Thalberg, Björn
A Keynesian model extended by explicit demand and supply functions for investment goods

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A KEYNESIAN MODEL EXTENDED BY EXPLICIT DEMAND AND SUPPLY FUNCTIONS FOR INVESTMENT GOODS

BY

BJÖRN THALBERG

ALMQVIST & WIKSELL

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PREFACE

In this study we shall work out and investigate a Keynesian model which does not relate investment to the rate of interest (or to the rate of interest and income) with merely a single function, but which describes the market for investment goods somewhat more thoroughly and explicitly. The point of reference is the central model of Keynes' General Theory, and it is hoped that the study may in some respects help to elucidate Keynes' model.

The extended Keynesian model below is based essentially on Keynes' assumptions, but it is somewhat less aggregated than Keynes' model. It is formulated more explicitly in terms of orthodox supply and demand analysis, describing separately the demand for investment goods, the supply of investment goods, the demand for consumption goods, and the supply of consumption goods. The role of the supply side may therefore appear more explicitly and clearly than it usually does in Keynesian models, where for one thing one operates with investment as a function of the rate of interest, i. e. "the marginal efficiency of capital schedule". This function or schedule is of a compound nature; many factors concerning both demand and supply are suppressed in its form. Thus, it is often doubtful to presume that its form remains unchanged when discussing the results of various changes of economic policy.

We shall work out the conclusions of our extended Keynesian model, and compare them with those of the central model of Keynes', considering such questions as: How will an increase in public investment, or a shift in the propensity to consume, affect total employment, or the price-level? The answers given by the extended model are more complex expressions, which are shown to contain the answers given by Keynes' central model as special cases. Or, in other words, the answers of the extended model qualify the answers of Keynes' central model. Thus, we are able to appraise in what situations it may be realistic to operate with an unchanged "marginal efficiency of capital schedule". Also, we shall go into the question of in what situations it may be justified to operate with the demand for consumption goods as a function of income without taking the distri-

bution of income between wages and profits into account. It may also be preliminarily suggested that the extended model, which can be said to include both the "multiplier" and, in a sense, also the "accelerator", provides an useful background for certain discussions concerning stability.

The appearance of yet another study dealing with Keynes' General Theory does not, of course, make the air electric with expectancy. My excuse for printing it — besides the great interest in Keynesian models and their widespread application — is that I have simply not seen a study very close to it. I hope that the study may interest sincere students of the Keynesian theory, since the discussion of the extended model may in some respects help to elucidate the implicit assumptions underlying Keynesian models, and since I also deal with the problem of the correct formal representation of Keynes' model.

A first version of this study was contained in a Cowles Foundation Discussion Paper No. 91, of May 10, 1960. I am grateful to Professor Tjalling C. Koopmans, who helped me to avoid some errors in Section 2, and to Professor James Tobin, who kindly read through the first draft and gave helpful comments. During the autumm of 1961 I revised, and to a certain extent elaborated, the original mimeographed version. My colleagues at "Institutionen för nationalekonomi" at the University of Stockholm have helped me in various ways with suggestions. In particular I must mention Ferdinand H. Banks, Franz Ettlin, Carl-Olof Klingberg, and Assar Lindbeck. More generally, I have benefited from Professor Trygve Haavelmo's lectures on investment theory at the University of Oslo. I am indebted to the Rockefeller Foundation for making possible studies and research in the United States. Further I am indebted to Statens råd för samhällsforskning (The Swedish Council for Social Science Research) for publication grants.

The second edition is mainly a reprint of the first edition, which appeared in March 1962. Section 4 is, however, extended with three pages.

Björn Thalberg

1. THE CENTRAL MODEL OF KEYNES' *GENERAL THEORY*[1]

We start off with a formal exposition of the central model of Keynes' *General Theory*, which will serve as reference point for discussions of our extended Keynesian model. By the "central" model of Keynes' *General Theory*, we mean the model which Keynes summarizes in *G.T.* Chapter 18 *The General Theory of Employment Re-stated*; the chapter in which he says that he will gather together the threads of his argument. We shall, for the sake of briefness, refer to this model as "Mod. K.".

There exists a number of diverging expositions of "the model of Keynes' *G.T.*" As an obiter dictum of this study we shall try to say something about what the correct formal exposition of this model is. We shall, therefore, be careful to tie our description of "Mod. K." to Keynes' own text, and to refer to it properly. For this reason we shall, to begin with, state "Mod. K." in *wage-units*, as Keynes actually did.

1 A. "Mod. K." in Wage-units

In our description of "Mod. K." we use the following symbols: Y_w, National income (per unit of time) in wage-units; C_w, consumption (per unit of time) in wage-units; I_w, investment (per unit of time) in wage-units; p_w, price level (the same for I- and C-goods) in wage-units; M_w, money supply in wage-units; w, money wage (of "an hour's employment of ordinary labour"); r, rate of interest level; N, total employment in hours of "employment of ordinary labour" (per unit of time).

Furthermore, as to the variables C_w, I_w, p_w and M_w, the symbol without subscript denotes nominal value (in money-units). Thus, C denotes nominal value of consumption, etc. But for the real value of national income we reserve the symbol Y. Nominal value of national income is expressed by pY. We have $Y_w = pY/w$, $C_w = C/w$, etc.

To measure a variable in "wage-units" thus simply means to deflate its

[1] John M. Keynes, *The General Theory of Employment, Interest and Money*. London 1936. We refer to this book as the *General Theory or the G.T.*

nominal value by w, i.e. the money-wage of "an hour's employment of ordinary labour". (w is Keynes' so-called "wage-unit", cf. *G.T.*, p. 41.)

Y_w is actually intended to represent *real* national income. When p and w are changing proportionally, it should make no difference whether we deflate by w or p. But this is, of course, not true in cases where w/p alters. Imagine, e.g., a case where the quantity (and quality) of production remain unchanged, and w remains unchanged, while p increases by 10 per cent. Y_w will then "register" an undue rise in real income of 10 per cent, whereas Y is constant. Keynes was aware of this weakness of his "wage-unit approach" (cf. *G.T.*, p. 91 below, p. 92 and p. 114,[1] and cf. below).

On the other hand, the use of w as a factor of deflation, instead of p, does—as we shall see—help to simplify Keynes' analysis.

Keynes' assumptions are, formally expressed:

(1.1) $Y_w = C_w + I_w.$

(1.2) $C_w = F(Y_w), \quad 0 < F' < 1.$

(1.3) $M_w = M/w = L(Y_w, r), \quad L_1' > 0, \quad L_2' < 0.$

(1.4) $I_w = f(r), \quad f' < 0.$

(1.5) w is fixed.

(1.6) M is fixed.

(1.7) $Y_w = pY/w = p\Phi(N)/w.$

(1.8) $\Phi'(N) = w/p, \quad \Phi''(N) < 0.$

(1.9) $N \leqslant N^s = g(w/p).$

Keynes considers a closed economy. Furthermore, he takes as given, *inter alia*, "the existing quality and quantity of available equipment, the existing technique".

We shall now briefly explain each of the above equations. (1.1) is an ecocirk relation, to use the expression of Ragnar Frisch. (It is not explicitly mentioned in Chapter 18, but it is carefully stated in Chapter 6 of the *G.T.*).

(1.2) is the familiar Keynesian consumption function. "Real" consumption is expressed as merely a function of "real" income. Though, Keynes does not argue that C_w is altogether independent of variations in the rate of interest. But as he claims that this influence is rather insignificant, we

[1] Here Keynes writes, *inter alia:* "In certain contexts we must not overlook the fact that, in general, Y_w increases and decreases in a greater proportion than real income; but in other contexts the fact that they always increase and decrease together renders them virtually interchangeable."

shall omit r as an argument in the F-function. Certainly, the level of cash balances, or variables describing *distribution* of income, do not—in Keynes' analysis—enter as arguments of the consumption function.

(1.3) expresses that the demand in wage-units for money holdings, $L(Y_w, r)$, shall equal the "real" money supply M_w. It is supposed that L_1', i.e. $\partial L/\partial Y_w$, is positive; and that L_2', i.e. $\partial L/\partial r$, is negative.

(1.4) expresses Keynes' investment function. It looks simple from the outside, but is of a very compound nature.

Let us, to simplify, treat productive capital as if it consisted of homogeneous units. (1.4) involves the "marginal efficiency of capital", which we shall denote by e, to be a declining function of the amount of new capital goods, ΔK, which the investors wish to purchase, i.e., $e = h(\Delta K)$, $h' < 0$. If we assume that the time of construction and delivery of the ΔK units of capital is fixed and equal to one time unit, it follows that (numerically) I equals ΔK.[1] Assuming further that the investors limit their contracting for new capital goods to the point where e, (which is $= h(\Delta K) = h(I)$) is equal to r, we get: $I = h^{-1}(r)$ or $f(r)$, where $f' < 0$.

Keynes suggests two reasons why e is a *declining* function[2] of ΔK; and, consequently, $f' < 0$. First, when ΔK increases "the prospective yield will fall". And second, when ΔK increases (the time of delivery being constant) "as a rule, pressure on the facilities for producing that type of capital will cause its supply price to increase" (*G.T.*, p. 136).

Keynes' investment function is sometimes interpreted as solely expressing current *demand* for I-goods. (As an example see Don Patinkin, *Money, Interest and Prices*, p. 130.) But the above-cited second reason, which

[1] The exposition will be complicated if we allow the construction time of the ordered capital units to be a variable, which the investors and the producers of I-goods have to decide upon. The criticism may be made that many authors just operate with I equal (or proportional) to ΔK without mentioning the simplifying assumptions involved. Also the *G.T.* (see p. 136) is insufficient on this point. Cf. here G. Arvidsson, "Några randanmärkningar till Keynes' investeringsteori", *Ekonomisk tidskrift*, no. 1, 1960. Arvidsson discusses *under what conditions* the "marginal efficiency of capital schedule", $h(\Delta K)$, coincides with the investment schedule $f(r)$.

In general, it is a deficiency of most explanations of the Keynesian investment function that they concentrate upon the question of how much the investors want to increase their stock of capital, and neglect to inquire how soon investors receive their new capital goods, although both types of information are needed to explain investment (and the employment of the I-goods industry).

[2] Recall that "the marginal efficiency of capital", e, is implicitly defined by the condition $q = \sum_i Q_i/(1+e)^i$, where Q_i is the prospective yield (in period $i = 1, 2, \ldots$) of the marginal unit of the new capital, and q is its supply price.

stresses the pressure on the facilities for producing capital, clearly shows that, in the reasoning of Keynes himself, Keynes' investment function also embraces the supply of I-goods. Actually, Keynes is using his concept "the marginal efficiency of capital" to compress the characteristics of both the demand and supply side of the I-goods market into *one* function, i.e. $I_w = f(r)$.[1]

We may conceive of several important factors, other than those mentioned by Keynes, which contribute to determine the numerical value of f', and to insure that $f' < 0$. Kalecki's principle of "increasing marginal risk" can help us to explain why the single investor limits his demand of new I-goods, even if he reckons that the price of his own products, and the price of I-goods, are fixed (independent of his own adjustment).[2]

Another, and fairly common, way of suggesting why f' is negative is that, considering all actual and potential investors, the number of profitable investment projects (i.e., projects which are expected to yield a rate of return which, expressed as a rate of interest, is $\geqslant r$) is ordinarily supposed to increase when we, *ceteris paribus*, imagine a lowering of the rate of interest r.[3] However, this increase may, for the actual range of r, be very slight during crises of confidence. Hence, a low numerical value of f' is often supposed to express a state of pessimistic price expectations. Further, if there is little or no excess capacity in the I-goods producing industry, an increase in ΔK would increase considerably the *pressure* on the facilities for *producing* capital, which will cause its supply price to increase (as Keynes writes). This, again, will, *ceteris paribus*, tend to give f' a low numerical value. (Cf. here also Goodwin's nonlinear accelerator.)

One can thus argue that Keynes' investment function takes several important factors into account. It reflects characteristics of both the demand and the supply side of the market for I-goods. It does this, however, *not explicitly*. — The extended model presented below represents an attempt to get behind Keynes' complex investment function.[4]

[1] In our extended model below, where we describe separately both the demand for and the supply of I-goods, we thus do not need to introduce the concept of the marginal efficiency of capital.

[2] M. Kalecki, *Essays in the Theory of Economic Fluctuations*, p. 95.

[3] Note, though, that there may be fallacies in *this* line of reasoning. I.a., to assume an *invariable* price of capital when we imagine a lower value of r; which is not in accordance with Keynes. (Cf. that the lowering of r tends to increase the demand for I-goods, and thereby also q.)

[4] If the investment function is (in contrast to Keynes) interpreted as solely expressing the *demand* for I-goods, it is, of course, somewhat less complex. However, there

In the *G.T.* Chapter 18 Keynes refers briefly to the equations (1.2)—(1.4) as "the three fundamental psychological factors, namely, the psychological propensity to consume, the psychological attitude to liquidity and the psychological expectation of future yield from capital assets".

While the price level is supposed to be *flexible*, the money wage rate is supposed to be *fixed*. Keynes' justification for this assumption is twofold. Firstly he argues that w actually tends to be rigid, at least in the downward direction. Secondly he holds that the assumption of a fixed money wage gives *analytical* advantages.[1] (1.6) assumes that the nominal supply of money is given, or more precisely, that M is fixed by central bank policy.

The assumptions (1.5) and (1.6) are clearly stated in Chapter 18, p. 247. Here Keynes says: "Thus we can sometimes regard our ultimate independent variables as consisting of (1) the three fundamental psychological factors [i.e. equations (1.2)–(1.4) above], (2) the wage-unit as determined by the bargains reached between employers and employed, and (3) the quantity of money as determined by the action of the central bank."

This sentence indicates, moreover, that Keynes regards equations (1.2)–(1.6) as constituting a determined set, whereby he can explain "real" national income Y_w. This is correct (when the definitional relationship (1.1) is included as well). "Mod. K." does really, when stated in wage-units, contain a determined subset, which enables Keynes to explain Y_w without taking the price level, or the form of the production function, into account. By the subset, (1.1)–(1.6), Y_w, C_w, I_w and r, are explained, and noteworthy from the demand side mainly. (1.7)–(1.8) then form a subset of a *higher order* to explain N and p. (Concerning causal ordering see Cowles Com. monograph no. 14, chapt. 3.)

"Mod. K" further includes an aggregate production function, $Y = \Phi(N)$, where N denotes the level of employment. The factors which Keynes considers as given (i.e. quality and quantity of equipment, existing technique, etc.) allow us, Keynes says (Chapter 18, p. 246), "to infer what level of national income measured in terms of the wage-unit will correspond to any given level of employment; so that, within the economic framework which we take as given, ... there is a unique correlation between the two".

According to (1.8), employers will employ workers until the marginal product of labor equals the real wage rate. Implicitly, (1.8) expresses the

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may be a tendency that expositions with such an interpretation of $f(r)$ neglect (more than others) to discuss the time of construction and delivery of the I-goods. Cf. footnote 1 on p. 11.

[1] As to this second point, cf. *G.T.* Chapter 19, especially p. 257.

demand for labor as a function of the real wage rate. In Chapter 18 the relationship (1.8) is mentioned on p. 249: "Increase in output will be accompanied by a rise of prices (in terms of the wage-unit) owing to increasing cost in the short period." (Cf. also *G.T.* Chapter 2, especially p. 17.)

The assumption that Φ'' is negative is quite central in Keynes' analysis. Imagining N to increase from a point where there is much excess capacity, $\Phi'(N)$ will, according to Keynes, decrease at first only slightly, but later more and more strongly as the "full capacity point" is approached. (*Cf. G.T.* p. 42, p. 296, and pp. 299–301.)

The aggregate production function does not furnish a direct relationship between Y_w and N. We get, however, a unique relationship between Y_w and N when both (1.7) and (1.8) are taken into account. Whenever N increases, Y_w increases. The increase in Y_w is, however, partly *nominal*. Because $\Phi'' < 0$ and w is constant, p and p/w will, according to (1.8), always increase when N increases. But Φ'' may be numerically very small, in which case p and p/w are approximately constant, and changes in Y_w describe a purely *real* increase in income.

Thus, when we consider Y_w as given, determined by equations (1.1)–(1.6), the variables N and p, and thereby also w/p, are determined by the equations (1.7)–(1.8).

(1.9) is a constraint which does not affect the degrees of freedom of the model. It says that the level of employment cannot *exceed* the supply of labor, which is (in the classical way) supposed to be a function of the real wage.[1] Incidentally, Keynes also applies the supply function of labor to define "full employment". $N = N^S = g(w/p)$, which in the *G.T.* is also referred to as the "second postulate of the classical theory", implies the absence of "involuntary" unemployment", Keynes writes. (See *G.T.* p. 15; and also p. 16 and p. 28.)

There should be no doubt that Keynes attached the constraint (1.9) to his model. This is stated in *G.T.*, p. 29, p. 246 and p. 292, but perhaps most clearly and explicitly on p. 30, where it says: "Thus the volume of employment is not determined by the marginal disutility of labour meas-

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[1] Conversely it is, according to "Mod. K.", possible that N is *less* than $N^S (= g(w/p))$. The workers, especially the unemployed, will, of course, not like such a situation. But, as Keynes says (p. 291), whilst labour is "*always in a position to refuse to work*" on a scale involving a real wage which is less than the marginal disutility of employment (i.e., $N > g(w/p)$), it is "*not in a position to insist on being offered work*" on a scale involving that $N = g(w/p)$.

ured in terms of real wages, except in so far as the supply of labour available at a given real wage sets a *maximum* level to employment."[1]

We may ask what would happen if the "effective demand" increases in a situation where $N = N^S = g(w/p)$, i.e., in a situation of full employment. In this case Keynes will apply a different model. He then no longer assumes that the money-wage rate is fixed and constant (cf. *G.T.*, p. 301 and also p. 29, point (6)). Instead he suggests (*G.T.*, Chapter 21) that we may then assume N to be constant.[2] Under these assumptions, an increase in the "effective demand" will raise p and w, but leave w/p and all real magnitudes unaffected.

1 B. "Mod. K." in Price-deflated-units

Keynes' idea of measuring quantities of money-value in terms of wage-units has not been applied much (if ever) by other authors. The reasons he gives (*G.T.*, Chapter 4) for his wage-units approach are not very convincing (but the *G.T.* was though, for one thing, written before the bulk of the literature on national accounting). Keynes intended to obtain expressions for real magnitudes by deflating nominal values by the money wage. Now, the usual practice is, of course, to apply an index for price level as a factor of deflation. We want, therefore, as a point of reference for our extended Keynesian model, a formal exposition of "Mod. K.", in which we differ from Keynes in the respect that p replaces w as a factor of deflation. We thus shall operate with, e.g., C/p instead of C_w, with I/p and M/p instead of, respectively, I_w and M_w; and with Y instead of Y_w (note that we let Y denote real national income and pY nominal national income).

We now state "Mod. K." in price-deflated-units:

(1.10) $pY = C + I.$

(1.11) $C = pF(Y),\ 0 < F' < 1.$

(1.12) $M = pL(Y, r),\ L_1' > 0,\ L_2' < 0.$

(1.13) $I = pf(r),\ f' < 0.$

(1.14) w is fixed.

(1.15) M is fixed.

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[1] Keynes' assumptions (1.5) and (1.9) emerge, by the way, very clearly in Modigliani's exposition, "Liquidity preference and the Theory of Interest and Money", *Readings in Monetary Theory*, p. 189.

[2] Alternatively, $N = N^S = g(w/p)$ may replace (1.5). If, though, the supply curve of labor is very steep, N would be approximately constant anyhow.

(1.16) $Y = \Phi(N), \Phi''(N) < 0.$

(1.17) $\Phi'(N) = w/p.$

(1.18) $N \leqslant N^S = g(w/p).$

The set of equations (1.10)–(1.17) looks much like the set (1.1)–(1.8). Four of the equations are exactly the same.

For convenience we use here the same symbols, i.e. F for the consumption function, L for the liquidity function and f for the investment function, as we did in the model (1.1)–(1.8) above. But this does *not* mean that the forms of the functions are identical (cf. below).

In the consumption function Y now enters as an argument instead of Y_w. This, I shall argue, is a better representation of Keynes' theory, relieving his concerns *G.T.* p. 91, 92. Compared with (1.2), the form of the consumption function in (1.11) has a *higher degree of autonomy*. The fact is that, according to Keynes, when Y_w increases, the increase is partly real and partly nominal. The extent to which this increase is nominal depends, as mentioned above, on the numerical value of Φ''. On the other side, the effect of a certain increase in Y_w on real consumption will (generally) depend upon how much of this increase is actually real.[1] In (1.2), therefore, the form of the consumption function is not *purely a matter of the consumption pattern* of the community, it depends upon other parts of the system as well. F in (1.11) is in this respect better, since an increase in Y is supposed to describe a purely real increase in national income.[2]

Also, we can in a similar way argue that f (the form of the investment function) in (1.13) possesses a higher degree of autonomy than f in (1.4). And further, one can argue that L in (1.12) has a higher degree of autonomy than L in (1.3).[3]

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[1] If the increase in Y_w is essentially nominal, F' in (1.2) may be close to unity even in cases where the marginal propensity to consume measured in real terms is low. Cf. the case of a linear consumption function, $C/p = \alpha Y + \alpha_0$ or $C = \alpha p Y + \alpha_0 p$, where—as usual—we assume that $0 < \alpha < 1$, and $\alpha_0 > 0$. Now, imagine an increase in nominal income (pY), which is essentially caused by a *rise in p*. In that case the relationship between the increase in C and the increase in (pY) is seen to *exceed* α; e.g., near the so-called "break-even point" $dC/d(pY)$—and thus also dC_w/dY_w—is close to unity independently of the value of α.

[2] As to the concept "the autonomy of an economic relation", cf. T. Haavelmo, *The Probability Approach in Econometrics*, Chapter 2.

[3] This may, however, be rather insignificant if the nominal demand for money to cover transactions and precautionary motives is approximately *proportional* to nominal national income. Then, to this particular demand, it does not matter whether an increase in income is real or nominal.

Thus, the transformation of "Mod. K." from Keynes' own presentation of it in terms of wage-units to a presentation of it in terms of p-deflated-units means in some respects an improvement. Also from the pedagogical point of view (1.10)–(1.17) is preferable to (1.1)–(1.8), as it feels rather clumsy to work with wage-units.[1]

On the other hand, "Mod. K." in wage units contains, as mentioned above, a determined subset. A fact which allows Keynes to *simplify* his analyses, as he can explain Y_w without having to explain, *inter alia*, the price level. The same is not true to the system (1.10)–(1.17). The four first equations of this system contain *five* unknown variables, viz. Y, C, I, p and r,[2] while the first four equations in (1.1)–(1.8) contain *four* unknown variables Y_w, C_w, I_w and r. Thus, when transferring "Mod. K" into p-deflated-units, as we did, Keynes' short-cut explanation of "real" income, which undoubtedly has been very valuable from the pedagogical point of view, gets lost. To explain Y we now have (unless Φ'' approaches zero) to go into the whole system and explain simultaneously p and N as well.

Incidentally, in the literature one finds many expositions of the "Keynes' model" which do *not* use the Keynes' wage-unit approach, but still explain real income without explaining the price level. This is, for example, nearly always the case in expositions in which the Hicksian IS-curve and LL-curve are applied (cf. Hicks, "Mr. Keynes and the 'Classics'", *Econometrica* 1937, and others). One operates, so to speak, as if the set of equations (1.10)–(1.13) contained only four unknown variables, because one tacitly assumes p to be constant. This way of proceeding is quite in accordance with Keynes, as long as one only considers situations with substantial unemployment and excess capacity (i.e. where Φ'' is approximately zero). But it violates Keynes' assumptions *whenever N is supposed to approach full employment* (cf. the fact that Keynes—because "resources are not homogenous" and "interchangeable"—presumes the numerical value of Φ'' to be quite significant "whilst there are still unemployed resources available"). (Cf. *G.T.*, p. 296 and pp. 299–301.)

[1] In his *Policy against Inflation* (London 1958), R. Harrod suggests that Keynes operates with wage-units in order "to keep clear the difference between a demand-pull and a cost-push inflation". What helps Keynes to keep clear this difference is, I think, rather his assumption of an *exogenously given* money-wage rate. I cannot see that "Mod. K." in wage-units (the equations (1.1)—(1.8) above) is in this respect any better than "Mod. K." in p-deflated-units (the equations (1.10)—(1.17) above).

[2] One may remark that, if (1.15) is altered such that M/p, and not M, is fixed (by central bank policy), the four first equations mentioned constitute a determined subset explaining Y, C/p, I/p and r.

In my opinion there should not be much disagreement about the formal exposition of the model Keynes summarizes in *G.T.*, Chapter 18. If one accepts (1.1)–(1.9), one will probably also accept (1.10)–(1.18) as "Mod. K." expressed in p-deflated-units.

Concerning (1.12), however, the formulation of the right-hand side of it is open to question, as Keynes did not express himself clearly at this point. Don Patinkin wonders whether the formulation $M = pL_1(Y) + L_2(r)$ is the correct interpretation of Keynes, because Keynes "never permits the speculative demand to absorb an increased supply of money except at a lower rate of interest". But, Don Patinkin concludes, this is probably due to "a simple—but vital—error", as Keynes "never explicitly pointed out that his speculative demand was independent of the price level" (see Don Patinkin, *Money, Interest and Prices*, p. 263 and 264). Undoubtedly, if Keynes really did have in mind that the speculative demand for money is independent of the price level p, it seems strange that he did not mention it explicitly. *On the other hand*, one may, as A. Lindbeck points out in a mimeographed paper ("Monetary and Fiscal Policy in an Aggregate General Equilibrium Model with Autonomous Tax Amounts", Stockholm 1960), argue that an assumption $M^D = pL_1(Y) + L_2(r)$ harmonizes pretty well with Keynes' *main lines* of reasoning. "The Keynesian model is characterized by a kind of dichotomy", Lindbeck writes (op. cit., p. II. 18), "For instance, a change in the preference for spending relative to saving use to be analyzed without assuming a corresponding change in the money demand function, and *vice versa*". Furthermore Lindbeck writes: "The speculative demand for money concerns speculation in bond prices. Hence, the liquidity preference function, to the extent that it concerns the demand for *speculative* cash, only deals with the choice between money and bonds. However, there is no reason why this choice should be influenced by commodity prices. The relation between real interest rate on bonds and the real interest rate on money is not affected by a factual or expected change in commodity prices."

Regarding our analyses below, however, the formulation of (1.12) is in fact not important because we mainly work on the assumption that r is autonomously given, instead of applying (1.12) and (1.15).

As regards (1.11), we could, in accordance with Keynes, relate C/p to both Y and r.

As to (1.13), one sometimes finds expositions of the Keynesian model, in which I/p is related to Y as well as to r. In my opinion this means an extension of "Mod. K.". I cannot see that Keynes ever, at least explicitly,

considered that I/p depends also on Y. I think one will find that expositions using $I/p = f(r, Y)$ lack direct references to the $G.T.$ on this point.[1]

Many authors emphasize that Keynes assumes some kind of "money illusion" on the part of the workers. One may ask whether such an illusion appears in our formal exposition of "Mod. K.". The labor market is described by (1.17) (which expresses the demand for labor), by the assumption (1.14) that w is fixed ("by the bargains reached between employers and employed") and by the constraint (1.18) which relates, in the classical way, supply of labor to the real wage rate. Actually, no money illusion is explicitly expressed in (1.10)–(1.18). Certainly, when Keynes discusses the assumption (1.14) of a rigid money wage, he gets into this question. "Every trade union will put up some resistance to a cut in money-wages, however small," he writes ($G.T.$, p. 15). But "no trade union would dream of striking on every occasion of a rise in the cost of living". Keynes does, however, claim that there is a rational behaviour behind this *apparent money illusion*. He stresses that there is, under a system of free wage-bargaining, "no means of securing a simultaneous and equal reduction of money-wages in *all* industries". Any individual or group, "who consent to a reduction in real wages relatively to others, will suffer a *relative* reduction in real wages, which is a sufficient justification for them to resist it" (see $G.T.$, p. 14, 15, 264, 265).—In fact it appears that the talk about the assumed money illusion in the labor market (or in the labor supply) in "Mod. K." is exaggerated or even misleading.

As an alternative to (1.15) Keynes sometimes operates with:

(1.15 b) $\qquad$ r is fixed by central bank policy.

If (1.15b) is applied, the five equations (1.10), (1.11), (1.13), (1.16) and (1.17)[2] form a *determined model* to explain the five unknown variables Y, C, I, p and N. This alternative and simpler model is indicated in Chapter 18, p. 245. "Our independent variables are, in the first instance, the pro-

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[1] Incidentally, Keynes' investment function does not take into account that the level of I/p in the present period depends also upon what happened in the previous periods, as the construction of many types of I-goods takes a long time. However, this fact is in general more essential in *dynamic* studies than in studies applying comparative statics. Anyhow, a constant term of the f-function may to some extent take care of this argument.

[2] If one still operates with (1.12), this equation will give us the level of M which is necessary to keep the rate of interest on its fixed level.

pensity to consume, the schedule of the marginal efficiency of capital and the rate of interest, though, as we have already seen, these are capable of further analysis." For an example in which Keynes applied this simpler version, see *G.T.*, pp. 260, 261. Below we shall in most cases employ or refer to this simpler version of "Mod. K.".

2. AN EXTENDED KEYNESIAN MODEL

As a point of deviation from "Mod. K." our extended model below describes separately, though briefly, the demand and the supply side of the market for I-goods, and the demand and the supply side of the market for C-goods. It thus may become possible to express more explicitly relevant characteristics of, *inter alia*, the demand and the supply of I-goods.

We shall make use of the theoretical construction called the "week", which, e.g., Don Patinkin applies in his *Money, Interest, and Prices*.[1] I.e., we assume that the acts of consumption, production and delivery are going on continuously, but that buyers and sellers meet only in the beginning of each "week". At this "meeting" the prices and quantities—for that "week"—are supposed to be determined in accordance with the demand and supply functions, and with the market clearance conditions. We describe thus a short-term equilibrium.

We consider first the demand for new capital goods by the producers of consumer goods. We assume homogeneous units of capital. For the sake of simplicity, we further assume that it takes a constant and given span of time, Θ "weeks", to construct, deliver and install any quantity of I-goods.[2] Thus, imagine a contract concluded this "week", i.e., "week" no. t_0, for the construction of Q capital units. This contract gives rise to an investment (or investment activity) of Q/Θ capital units *per "week"* from the beginning of week no. t_0 to the beginning of week no. $(t_0 + \Theta)$.

The production per "week" of the consumption-goods producers depends upon their inputs of labor and capital. At the beginning of the "week" t_0 the size of their capital stock is a historically given datum. So also is the

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[1] Cf. *Money, Interest and Prices*, pp. 8–9 and p. 37. Application of this construction is, I think, convenient to describe a Keynesian short-term equilibrium where one takes into account the income-generating, but not the capacity-generating effect of current investment activity. "Week" suggests, by the way, too short a period; "quarter" (of a year) would be better. However, it is rather not appropriate in our *static* analysis to define the length of the "week" sharply. Interpretations may vary.

[2] I.e., we assume that the producers deliver the amount of capital goods contracted for in the beginning of this "week" *all at the same time* after Θ time units. To secure that the level of production will be constant throughout each "week", we will assume that Θ *is an integer*, and that new capital is put to use at the beginning of the "week".

number of capital units they have already ordered, but not received. Let K_0 denote their total given stock of capital, *including* what is ordered, but not yet delivered. If the C-goods producers contract at t_0[1] for a number of Q_c capital units, their stock of capital in "week" $(t_0 + \Theta)$ will be $(K_0 + Q_c)$.[2] Their total planned input of labor at $(t_0 + \Theta)$ we denote by n_c. Their total planned output per "week" from $(t_0 + \Theta)$ is consequently $f(K_0 + Q_c, n_c)$, or $G(Q_c, n_c)$, since K_0 is predetermined. We assume G''_{11} and G''_{22} to be negative[3] (i.e., diminishing returns to each single factor). We assume that the producers of C-goods, when deciding how much new capital they will contract for this week, act as if they maximize:

$$V = \int_{\Theta}^{\infty} [(pG(Q_c, n_c) - wn_c] e^{-r\tau} d\tau - qQ_c. \tag{2.1}$$

V expresses the present value of their expected income[4] less the outlay qQ_c. The symbol p denotes the price level of consumption-goods, and w denotes the nominal wage rate level. r denotes the existing level of the rate of interest (we thus assume that the producers of C-goods will want to get at least this rate of interest on their capital outlay). q denotes the price level of I-goods. We shall assume that each producer considers the price and wage level as given, *independent* of his *own* supply and demand.[5] In other words, our producers are supposed to be "quantity adapters". Furthermore, we shall operate with the simple assumption that the producers expect p and w to *remain constant* (on the levels existing this "week").[6]

It is assumed that the purchasers of I-goods pay at the time they contract, even though they have to wait Θ time units for the delivery. This is, of course, a particular assumption, even if it is perhaps usual to pay

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[1] I.e., in the beginning of "week" no. t_0.

[2] To abstract from the factor of depreciation, we assume that capital goods retain full technological efficiency, and constant maintenance costs, indefinitely.

[3] G''_{11} denotes the second-order derivative of $G(Q_c, n_c)$ with respect to Q_c, and G''_{22} the second-order derivative with respect to n_c.

[4] Since we imagine capital to last forever, the expected income stream is integrated from Θ *to infinity*. Of course, a more realistic assumption is to reckon with a certain life-span of capital, e.g. of h "weeks", from delivery to abrupt and total obsolescence, i.e., the so-called "one-hoss-shay" assumption. Here we use ∞ as an "approximation" for h. This simplification does not, I think, influence the core of later arguments.

[5] Because we imagine a competitive market where the acting units are all relatively small.

[6] This assumption of "horizontal" or "static expectations" is common, particularly, of course, in *static* analyses. "Horizontal" expectations are also suggested in the *G.T.*, Chapter 12. We return to the question of price expectations in Section 4, p. 52, below.

part of the sum when the contract is made. (Of the two simple alternatives to assume payment at the time of ordering or at the time of delivery, I have chosen the former because it gives perhaps slightly simpler formulae. For the conclusions of our comparative statics studies of Section 3 it does not, as a rule, matter which of the two alternatives we choose.)

A necessary condition for maximum of V (at some $Q_c > 0$) is that its partial derivative with respect to Q_c be zero. This condition gives:

$$p\,G_1'(Q_c, n_c) = r\,q\,e^{r\Theta}. \tag{2.2}$$

(2.2) expresses the condition that the expected income of the marginal unit of capital shall equal its price times the rate of interest times $e^{r\Theta}$.[1]

Another necessary condition for maximising V is that its partial derivative with respect to n_c be zero, which gives:

$$p\,G_2'(Q_c, n_c) = w. \tag{2.3}$$

(2.3) expresses the condition that planned input of labor at $(t_0 + \Theta)$, when the Q_c capital units purchased at t_0 are installed, shall be so high that the marginal product of labor will equal the expected real wage rate.

Actually, we are not interested in the variable n_c *per se*. As mentioned, n_c denotes the planned input of labor in the "week" $(t_0 + \Theta)$. Thus, n_c does not directly affect the demand for labor *this* "week". But n_c affects the future gains of capital, in the sense that it affects the marginal product of capital, i.e. $G_1'(Q_c, n_c)$, at $(t_0 + \Theta)$. This in turn affects the ordering of capital goods this "week", and thereby this "week's" employment in the I-goods industry. Therefore, in order to explain Q_c, and our other endogenous variables, we have also to explain n_c. Only if we especially assume that G_{12}'' is zero for all actual values of Q_c and n_c need we not include n_c, and equation (2.3), in our model.

We shall assume that at the point of adjustment $G_{11}''\,G_{22}'' > (G_{12}'')$.[2] In that case (2.2) and (2.3) give a maximum of V. If this condition is not met, no values of Q_c and n_c yield maximum V, given the values of p, w, q and r. This assumption is plausible, I think. When investors consider large increases in their stock of capital within the limited period of Θ "weeks", they must consider the problem of getting labor with the same degree of skill, and other difficulties.[2]

[1] The term $e^{r\Theta}$ enters because we assume that the purchasers pay at the time of ordering. This is seen to tend, *ceteris paribus*, to lessen the demand for capital goods.

[2] For many firms profitable production possibilities are quite limited, i.e. *decreasing returns to scale* prevail distinctly. A few firms may, however, see extremely good

(2.2) and (2.3) describe equilibrium conditions of a type familiar in analyses of the firm assuming perfect competition. We have, however, dealt with *total* production, etc., without going into *aggregation* problems. It may briefly be mentioned that the function $G(Q_c, n_c)$ is, of course, supposed to possess characteristics which are typical of the individual production functions, e.g., diminishing returns to a single factor. Furthermore, when we consider an equilibrium situation, and consider equilibrium prices as given parameters, we may say that the single firms—by maximizing their own profit—act as if they jointly and directly maximize the profit of the whole industry.

(2.2) and (2.3) give us Q_c and n_c as functions of p, q, w and r (or more precisely of p/q, p/w and r). The demand for I-goods by the producers of C-goods is thus described.

Another group demanding I-goods is the producers of I-goods themselves. The number of capital units which this group orders this "week" we denote by Q_i. Their planned labor input at $(t + \Theta)$ we denote by n_i. Their total planned output per "week" from $t_0 + \Theta$ is given by their production function, $H(Q_i, n_i)$.

We assume that the producers of I-goods, when they decide how much new capital they will demand this "week", act as if they maximize:

$$V_i = \int_{\Theta}^{\infty} (qH(Q_i, n_i) - wn_i)\, e^{-r\tau}\, d\tau - qQ_i. \tag{2.4}$$

V_i expresses the present value of expected income (cf. the expression V in (2.1) and the assumption that I-goods are paid at the time of ordering). We assume that each producer considers q, w and r to be independent of his own demand and supply, and furthermore that they expect q, w and r to *remain constant* on the levels existing this "week".

Necessary conditions for a maximum of V_i (at some $Q_i > 0$) are that its partial derivative with respect to Q_i, and with respect to n_i, be zero. Which gives:

possibilities, and therefore plan to expand immensely. But in such cases they usually plan to expand *in steps*, due to the difficulty in getting skilled workers, and the fact that some time is required to gain new and necessary experience, etc. As an important factor we may also stress Kalecki's "principle of increasing marginal risk" (cf. p. 12 above). The form of the G-function can implicitly express several of these factors which make the individual producer limit his immediate ordering of I-goods, even when his price expectations are horizontal (and independent of his own supply and demand).

(2.5) $$H_1'(Q_i, n_i) = r\, e^{r\Theta}.$$

(2.6) $$H_2'(Q_i, n_i) = w/q.$$

We assume that H_{11}'' and H_{22}'' are negative (diminishing returns to a single factor). Further, we assume decreasing returns to scale, or more precisely, $[H_{11}'' H_{22}'' - (H_{12}'')^2]$ is supposed to be positive (at the point of adjustment) (cf. here footnote 2 on page 23).

The equations (2.5) and (2.6) describe Q_i and n_i as functions of w/q and r. Note for one thing that an increase in the price of capital goods will not stimulate Q_i directly, i.e. not "through" (2.5). (A higher q means that the producers of capital get better paid, but, on the other hand, it also makes it more expensive for them to expand their plants.) Only in so far as q increases in relation to w will they increase n_i, and consequently Q_i, if $H_{12}'' > 0$ (i.e. complementary factors). In the case H_{12}'' is zero a change in q will, *ceteris paribus*, not affect the demand for I-goods by the actual producers of such goods.

We now turn to the *production side* of I-goods. When at the beginning of "week" t_0, the producers of I-goods are meeting the (potential) buyers of I-goods, they are already obliged to fulfill a certain number of previously concluded contracts. These obligations bound them, we assume, to produce Φ_1 capital units per "week" (in "week" t_0).

Secondly, the I-goods-producing sector also carries out public investment programs. For this purpose we assume that the I-goods producers are obliged to produce Φ_0 capital units per "week" (in "week" t_0). We will look upon Φ_0 as an economic policy parameter. An increase in Φ_0 expresses a higher level of public investement.

The sum $(\Phi_0 + \Phi_1)$ we denote Φ^0. As Φ_1 is supposed to be predetermined (i.e. a constant), an upward shift in Φ^0 is supposed to express an increase in the level of public investment.[1]

The number of workers which the I-goods producers employ during the "week" t_0 we denote as N_i. As their stock of capital during this "week" is a predetermined datum,[2] we write their aggregate production function as $\Phi(N_i)$.[3]

Out of the production flow of $\Phi(N_i)$ capital units per "week", previously

[1] If, particularly, $\Theta = 1$, Φ_1 is zero and $\Phi^0 = \Phi_0$.

[2] Cf. footnote 2, p. 21, above.

[3] Φ (N_i), "the production function with *present* equipment", is related to the production function H (Q_i, n_i), which enters in (2.4). They are, however, not identical.

concluded contracts and public investment programs together require Φ^0 capital units per "week". The flow of production which is left over to meet the demand for I-goods (by the private sector) this "week" is thus $(\Phi(N_i) - \Phi^0)$. This means a supply of $(\Phi(N_i)) - \Phi^0)$ capital units *per* "week", i.e. a level of production which corresponds to a number of $(\Phi(N_i) - \Phi^0)\,\Theta$ capital units delivered at $t_0 + \Theta$.

We have, as an equilibrium condition, that the supply of I-goods (for the private sector) shall equal the demand:

$$(\Phi(N_i) - \Phi^0)\,\Theta = Q_c + Q_i.\text{[1]} \tag{2.7}$$

The producers of I-goods calculate, we assume, that the orders they are concluding to-day give them a gross income per "week" of:

$$q\,[\Phi(N_i) - \Phi^0]. \tag{2.8}$$

By differentiating (2.8) with respect to N_i we get $q\,\Phi'(N_i)$. This expresses the marginal income of the labor input in the I-goods-producing sector. On the ground of profit maximization we suppose that the income of the marginal hour of labor input equals the money wage rate.

$$q\,\Phi'(N_i) = w. \tag{2.9}$$

We suppose that Φ'' is negative. Thus (2.9) implies, *inter alia*, that the supply of I-goods increases when q/w increases.

We now turn to the *market for consumption goods*. The production of the C-goods-producing industry, Y_c, is supposed to depend upon its labor input, N_c:

$$Y_c = \Psi(N_c). \tag{2.10}$$

This applies to the production in the "week" no. t_0. The size of the capital equipment is constant throughout the "week". (Cf. our assumption that Θ is an integer; footnote 2, p. 21.)

We assume that Ψ' is positive, and Ψ'' negative, i.e., we assume decreasing returns (in the single factor N_c).

There is a certain relationship between $G(Q_c, n_c)$ and $\Psi(N_c)$. The latter function we can derive from the former by, *inter alia*, putting Q_c equal to zero and n_c equal to N_c. (I have, though, found it convenient to operate

[1] (2.7) can also be explained this way. The ordering of Q_c capital units gives rise to an investment level of Q_c/Θ capital units per "week", and the ordering of Q_i capital units gives rise to an investment level of Q_i/Θ. The level of production by which the I-goods producers are to match this demand is $(\Phi(N_i) - \Phi^0)$ capital units per "week".

with two different function-symbols. Cf. (p. 22 above) the fact that K_0 includes capital which is ordered but not yet delivered.)

The calculated net income per "week" of the producers of C-goods is $(p\Psi(N_c) - wN_c)$. A necessary condition for maximum net income is that its derivative with respect to N_c be zero, which gives:

$$p\Psi'(N_c) = w. \tag{2.11}$$

(2.11) assumes that the producers of C-goods will employ workers until the marginal product of labor equals the real wage, w/p.

In order to simplify our analysis we want to *exclude real income* from our list of variables. The reason is that we allow the price levels of C-goods and I-goods to vary in different proportions, which complicates the definition of real income. Consequently, we cannot work with the usual Keynesian assumption that the real demand for C-goods, Y_c^D, depends directly upon the level of real income.

Instead we shall (provisionally) simply assume that Y_c^D depends upon the level of total employment, i.e. $Y_c^D = g(N_i + N_c)$. This is close to Keynes' assumptions anyhow, since "Mod. K." (1.10–1.18) assumes that $C/p = F(Y)$, and that $Y = \Phi(N)$, hence that $C/p = F(\Phi(N))$ or $g_1(N)$. But comparing g_1 with F, the form of the original consumption function, F, has a higher degree of autonomy.

Actually, the degree of autonomy of our function $g(N_i + N_c)$ (and of the Keynesian consumption function (1.11) as well) tends to be low *also* for the reason that the *distribution of income* between wages and profits is not explicitly taken into account. Note that this distribution will depend upon the *form* of *other* relationships of our model, such as $\Psi(N_c)$ and $\Phi(N_i)$. In order to improve the degree of autonomy of our consumption function considerably we should also seek to include the distribution of income between wages and profits. Such an attempt is made in Section 4 below.

As an equilibrium condition we have that the supply of C-goods is to equal the demand for them, i.e. $Y_c = Y_c^D$, or

$$Y_c = g(N_i + N_c). \tag{2.12}$$

We assume that g' is positive. We may ask if there is a parallel to the familiar and crucial assumption of Keynes' consumption function (1.11), that the marginal propensity to consume is less than unity. As it appears from our discussions below, and as one may intuitively concede, an assump-

tion that the fraction g'/Ψ' be less than unity, takes care of essentially the same thing.

The nominal supply of money we denote by M. We assume that M is autonomously given by central bank policy, i.e., $M=\overline{M}$. As an expression for the *real* money supply we write $\overline{M}/p$. The real demand for money holding is supposed to be a function of total employment and the rate of interest, i.e., $L[(N_i+N_c), r]$. Here the argument (N_i+N_c) is supposed to express the demand for money holding caused by the transactions motive, while the argument r is connected with the speculative motive.

As an equilibrium condition, we have that the supply of money shall equal the demand for it:

$$\overline{M}/p = L[(N_i+N_c), r]. \tag{2.13a}$$

As an alternative to (2.13a) we may operate with the assumption that the rate of interest is directly fixed, determined autonomously by the monetary authority.

(2.13b) r is given.

Such an assumption will, as compared with the alternative (2.13a), often simplify our analysis considerably.—(We consider only $r>0$.)

We finally assume that the level of the money wage rate is given, predetermined by the bargains reached between employers and employed. (While the prices p and q are supposed to be flexible.)

If we assume that r is fixed, the model (2.2), (2.3), (2.5)–(2.7), (2.9)–(2.12) above provides nine independent equations to explain the nine variables p, q, Q_c, Q_i, n_c, n_i, N_c, N_i and Y_c. If we alternatively operate with (2.13a), r will also be a variable, and the model provides ten equations to explain ten variables.

In "Mod. K." we operated with one aggregated production function for both I- and C-goods, and with a single variable describing the general price level (as prices of the two types of goods mentioned were supposed to vary in the same proportions). Actually, we may say that there is a certain *asymmetry* in the way Keynes treats investment and consumption. In "Mod. K." (1.16) describes the supply of both I- and C-goods, (1.11) describes the *demand* for C-goods, but (1.13) does not describe just the demand for I-goods. (1.13) states the level of I/p, given r. As mentioned, both characteristics of the demand and the supply side affect the form of the function $f(r)$.

In our extended model we operate more symmetrically with separate markets for I-goods and C-goods. As an equilibrium condition, demand equals supply in each market. By this elaboration of "Mod. K." we are, to some extent, able to go behind Keynes' complex investment function.[1] Otherwise, however, the assumption of our extended model are essentially quite similar to those of the "Mod. K.". Comparing the equations, (1.13) of "Mod. K." is related to the six equations (2.2), (2.3), (2.5)–(2.7) and (2.9) of the extended model. (1.17) is related to (2.9) and (2.11). Furthermore, (1.11) is related to (2.12), and (1.12) and (1.15) are in the extended model expressed by (2.13a). In both models the money wage is supposed to be given and constant. The constraint (1.18) of "Mod. K." also applies to the extended model, though it has not hitherto been mentioned in Section 2. In all cases we will disqualify our analysis if we find that total employment exceeds the total labor supply. It should be noted, however, that in our extended model a partial or total scarcity of labor may (cf. G. T. p. 42) affect the numerical values of Φ'' and Ψ'', whereby a violation of (1.18) may be hindered (granted that certain stability conditions are met).

Both "Mod. K." and the extended model can describe situations of *equilibrium with unemployment*. Our assumptions do not remove the possibility that total employment is *less* than total labor supply $N^S (= g\ (w/p))$. Indeed, we may imagine situations where the potential investors are satisfied with the amount of capital they possess (including what is ordered but not yet delivered), or even where they find that they have too much capital. In these situations N_i will tend to be zero. However, if such a tendency arises it may more or less be counterbalanced by a fall in q.

In the next section we proceed to compare the answers provided by the two models to the question of how employment and prices will respond to various changes (e.g., of economic policy).

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[1] Going behind Keynes' complex investment function are also a number of expositions applying Lerner's concept the "marginal efficiency of investment" (MEI). However, it seems to me that these expositions are rather unclear on several points, what is for one thing assumed about the time of construction and delivery of capital goods? Furthermore, to speak about the marginal efficiency of *investment* is—because of the dimensions involved—strictly seen bizarre; one produces by means of factories, not by means of factories *per unit of time*. The alleged weakness of the MEI-expositions may be ascribed to the fact that they concentrate excessively on the use of diagrams, and rather disregard analytical formulation.

3. A COMPARISON OF THE CONCLUSIONS FROM THE TWO MODELS

By means of comparative statics we can, within our two models, discuss how total employment and other endogenous variables will respond to different shifts in parameters. In this section we intend to discuss thoroughly the effects within the two models of a change in the level of public investment. We will also, more briefly, look into the effects of shifts in the propensity to consume, in the rate of interest, in the money wage rate and in the nominal money supply. Finally, we shall briefly try to generalize how the results from the two models compare.

3 A. A Change in the Level of Public Investment

We consider first "Mod. K." (1.10–1.17). The investment function (1.13) we will now write $I/p = f(r) + I_0$. A shift in the level of the constant term I_0 can be interpreted as an autonomous shift in the level of public investment. The investment function together with (1.10) and (1.11) give $Y = F(Y) + f(r) + I_0$. We assume that the rate of interest is fixed and constant, i.e., we operate with (1.15b) and not with (1.15). Differentiating with respect to I_0 then gives:[1]

$$\frac{dY}{dI_0} = \frac{1}{1 - F'}. \tag{3.1}$$

Using the equations (1.16) and (1.17) we further get:

$$\frac{dN}{dI_0} = \frac{1}{1 - F'} \frac{1}{\Phi'}. \tag{3.2}$$

$$\frac{dp}{dI_0} = -\frac{1}{(1 - F')} \frac{\Phi''}{\Phi'} \frac{p^2}{w}. \tag{3.3}$$

(3.1) and (3.2) express the familiar Keynesian multiplier effect. The lower the marginal propensity to save $(1 - F')$, the stronger is this effect. (3.3)

[1] An alternative way of proceeding is as follows. The consumption function may be written $C/p = F(Y) + C_0$. A shift in C_0 may be interpreted as a shift in the level of public investment. In this case we get $Y = F(Y) + C_0 + f(r)$. Differentiating with respect to C_0 leads obviously to the same results.

shows the effect of an increase in I_0 on the price level. An increase in I_0 is seen to increase p. If the numerical value of Φ'' is high, i.e., if we are close to the "full capacity point", p may change considerably, even if the multiplier $1/(1 - F')$, and thereby the effect on Y, is quite small.

In the above analysis we have disregarded the possibility that changes in the level of public (real) investment may directly and/or indirectly affect the level of private (real) investment. When r has a fixed value, the level of private real investment was supposed to be given (equal to $f(r)$) independently of the value of I_0.[1]

We now turn to the extended Keynesian model of Section 2 above. We shall apply (2.13b), and thus not (2.13a), i.e., we operate with the version which assumes r to be fixed. Our point of reference is, therefore, the version of "Mod. K.", where r is fixed.

A closer look at our extended model reveals that it takes into account several forces whereby changes in the level of *public* investment, Φ^0, can affect the level of *private* investment. *Intuitively*, when public investment increases, the I-goods-producing sector applies more of its available resources for these purposes, and it *may* thus tend to withdraw part of its supply intended for the private sector. Consequently, according to the law of supply and demand q tends to rise, i.e., it becomes more expensive for private investors to purchase I-goods. This suggests a negative influence on Q_c (of an increase in Φ^0). However, the fact that q rises might actually stimulate the demand for I-goods by the very *producers* of these goods. Moreover, an increase in Φ^0 will, through the multiplier, tend to increase the demand for consumption goods, whereby p may rise. As we assume horizontal price expectations, based on the existing level of p, a rise in p will, *ceteris paribus*, increase the demand for I-goods by the producers of C-goods.[2]

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[1] The more usual version of "Mod. K.", in which the nominal money supply, and not the rate of interest, is given, involves an increase in *public* investment having a negative effect on the level of *private* investment. This is because higher values for Y and p, initiated by the upward shift in public investment and furthered by the "multiplier", increase the demand for money for transaction purposes, whereby r rises and private investment, I, falls. The fall in I will not, however, go so far that it prevents *total* investment $(I + I_0)$ from rising (when I_0 shifts upwards). Thus according to the more usual version of "Mod. K.", a rise in I_0 increases Y and p, but not to the extent described by (3.1) and (3.3). Note that when we differentiate the abovementioned more usual version of "Mod. K."—where (1.13) now is written $I/p = f(r) + I_0$—with respect to I_0, both dY/dI_0 and dr/dI_0 are seen to be positive.

[2] When (2.13a) is applied, i.e. we assume M (and not r) to be fixed; we furthermore

Now, according to (2.7), $Q_c = (\Phi(N_i) - \Phi^0)\Theta - Q_i$. Further, according to (2.11), $p = w/\Psi'(N_c)$. And according to (2.9), $q = w/\Phi'(N_i)$. Inserting these expressions for Q_c, p and q into (2.2) and (2.3), we get:

$$\text{(a)} \qquad \Phi'(N_i)G_1'\{[(\Phi(N_i) - \Phi^0)\Theta - Q_i], n_c\} = \Psi'(N_c)re^{r\Theta},$$

$$\text{(b)} \qquad G_2'\{[(\Phi(N_i) - \Phi^0)\Theta - Q_i], n_c\} = \Psi'(N_c).$$

(2.10) and (2.12) furnish a third equation:

$$\text{(c)} \qquad \Psi(N_c) = g(N_i + N_c).$$

Let us, to begin with, assume that H_{12}'' is zero (for actual values of Q_i and n_i). According to (2.5), Q_i will then be constant, independent of variations in Φ^0 (cf. p. 25 above). In such a simplified case, when we do not have to take into account changes in the demand for I-goods by the producers of I-goods themselves, the three equations (a), (b) and (c) form a determined model to explain the three variables N_c, n_c and N_i.

We will now discuss how a change in Φ^0 will affect N_i and N_c. Thereby we also learn how total current employment $(N_i + N_c)$ is affected.

Let us first look into the case where G_{12}'' (Q_c, n_c) is zero.[1] In this case changes in N_i and N_c are explained by the equations (a) and (c). Differentiating (a) and (c) with respect to Φ^0 we get:

$$(3.4) \qquad \frac{dN_i}{d\Phi^0} = \frac{(\Psi' - g')\Theta\,\Phi'\,G_{11}''}{(\Psi' - g')[G_1'\Phi'' + \Theta(\Phi')^2 G_{11}''] - re^{r\Theta}g'\Psi''}.$$

$$(3.5) \qquad \frac{dN_c}{d\Phi^0} = \frac{\Theta\,\Phi'\,g'\,G_{11}''}{\text{Denominator as in (3.4)}}.$$

To ease the interpretation we shall at first consider a case where the numerical values of both Φ'' and Ψ'' approach zero. This assumption may be realistic in a very *depressed economy* with plenty of excess capacity in the production of both I-goods and C-goods. In this case (3.4) simplifies to:

$$(3.6) \qquad \frac{dN_i}{d\Phi^0} = 1/\Phi'(N_i).$$

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take into account that with higher levels of production and prices (initiated by an increase in Φ^0) r tends to increase too, which, *ceteris paribus*, tends to reduce Q_c and Q_i somewhat.

[1] We then need not take into consideration the effects of a planned change in input of labor, together with the Q_c units increase in the input of capital, at the point of time $t_0 + \Theta$.

(3.6) involves that an expansion of public investment, Φ^0, increases the employment of the I-goods-producing sector just as much as is needed for the said expansion of Φ^0, and *leaves private investment unaffected.*[1]

Under the same circumstances (3.5) reduces to $g' / (\Psi' - g')$ times $1/\Phi'$. The effect on *total* employment, $(N_i + N_c)$, therefore approaches:

$$\frac{d(N_c + N_i)}{d\Phi^0} = \frac{\Psi'}{(\Psi' - g')} \frac{1}{\Phi'}. \tag{3.7}$$

We assume that $(\Psi' - g')$ is positive, i.e., when, *ceteris paribus*, one more worker is employed in the C-goods producing sector, the consequent increase in the output of C-goods will exceed the consequent increase in the demand for C-goods. In other words, the worker is supposed to save part of his income.

The right-hand side of (3.7) can also be written $1/\Phi'$ times $1/(1 - g'/\Psi')$. As, on p. 27 above, we interpreted g'/Ψ' as expressing something similar to the marginal propensity to consume (i.e. F' in "Mod. K."), we may interpret $1/(1 - g'/\Psi')$ as our "multiplier".

Comparing (3.7) to (3.2) we thus see that under the particular assumptions given above our extended model gives a conclusion very similar to that of "Mod. K.". According to (3.2), the lower the marginal propensity to save $(1 - F')$, the more an increase in public investment will stimulate employment; the "multiplier" being $1/(1 - F')$. In (3.7) the "multiplier" is $1/(1 - g'/\Psi')$. In both cases the original increase in the employment of the I-goods sector stimulates the demand for C-goods (and N_c), which again stimulates the demand for C-goods, etc., but without any feedback effect on investment. Also, in both cases the system "explodes" if, respectively, F' or g'/Ψ' approach unity.

It is interesting to notice the kind of restrictive assumptions which permit our extended model to give about the same simple conclusions as "Mod. K." gives. We assumed that Ψ'' and Φ'' approach zero,[2] an assumption which seems to be realistic in cases of a *deep depression*, i.e. the state of affairs which Keynes chiefly had in mind when he wrote his *G.T.* In such cases an increase in Φ^0 stimulates employment without raising p and q. Thus, there is neither an increase in p to induce the *producers of C-goods* to demand

[1] Note that N_i has to be raised by $1/\Phi'$ to secure, *ceteris paribus*, an increase in Φ^0 of one unit.

[2] We also assumed H_{12}'' and G_{12}'' to be zero. But these assumptions are actually not essential in the case $\Psi'' \equiv \Phi'' \equiv 0$ (whereby p and q tend to be constant). Cf., *inter alia*, (2.6) and (2.3).

more I-goods, nor is there any increase in q which tends to reduce their demand for I-goods. Nor is there any increase in q to induce the *producers of I-goods* to increase their own demand for I-goods. In such a situation it seems legitimate to operate with a "marginal efficiency of capital schedule", $f(r)$, which *does not shift.*

Above we assumed a situation with mass unemployment and plenty of excess capacity in *both* the I-goods- and the C-goods-producing sector. Let us now look into a case where the "ceiling" is quite close in the C-goods sector, but where there is still plenty of excess capacity for the production of I-goods. We can imagine such an equilibrium if we assume that resources which are capable of being transferred from the one sector to the other in the *short run* are very limited. (Thus, we also assume that p/w may differ from q/w.) Cf. here Keynes (*G.T.*, p. 296): "Since resources are not interchangeable, some commodities will reach a condition of inelastic supply whilst there are still unemployed resources available for the production of other commodities."

We assume, thus, that Φ'' approaches zero while the numerical value of Ψ'' is considerable. In this case (3.4) simplifies to:

$$(3.8) \qquad \frac{dN_i}{d\Phi^0} = \frac{(\Psi' - g')\Theta\,\Phi'\,G_{11}''}{(\Psi' - g')\,\Theta\,\Phi'\,G_{11}''\,\Phi' - re^{r\Theta}g'\,\Psi''}.$$

In (3.8) the sign of the numerator is negative (as G_{11}'' is negative). Thus, the first term of the denominator is also negative. However, the second term $(-re^{r\Theta}\,g'\,\Psi'')$ is positive (as Ψ'' is negative). We shall assume that the numerical value of the first term of the denominator exceeds that of the second term. This is, I think, a necessary condition for the stability of our equilibrium.[1]

Intuitively, the situation *can* be the following. The given increase in Φ^0 initiates a rise in p, which induces the producers of C-goods to order more capital. This, in turn, brings about an increase in N_i, but not in q (if Φ'' is

[1] Imagine a case where $|G_{11}''|$ is relatively small and where $|\Psi''|$ is very high, such that the denominator of (3.8) is positive. Imagine that, in a possible equilibrium situation, Φ^0 shifts upwards. Intuitively; N_i rises initially, which will produce a tendency for a rise in N_c. A rise in N_c necessitates a sharp rise in p (cf. (2.11)). The price of I-goods q remains constant, as $\Phi'' = 0$. The rise in p will, therefore, disturb the equality (2.2) $p\,G_1' = r\,q\,e^{r\Theta}$. Can a rise (or a fall) in Q_c restore this equality? Not in this case. A stable equilibrium seems not plausible in this case. (Even a very *small* increase of Φ^0 will make Q_c, N_i and p "explode", while a lowering of Φ^0 initiates what Keynes calls a sudden collapse of the "marginal efficiency of capital schedule".)

zero, cf. (2.9)). The rise in N_i stimulates a further increase in p, etc. No new equilibrium is reached, as long as $|\Psi''|$ is very high in relation to $|G_{11}''|$ and $|\Phi''|$,[1] i.e. so high as to make the denominator of (3.8) positive. How high $|\Psi''|$ can be in relation to $|G_{11}''|$ and $|\Phi''|$ depends, *inter alia*, upon $(\Psi' - g')$.

When the denominator of (3.8) is negative, (3.8) says that N_i increases when public investment is increased. The increase in N_i (per unit increase of Φ^0) is seen to exceed $1/\Phi'$, which means that not only total, but also *private*, investment increases (cf. footnote 1, p. 33). This is due to our "built-in accelerator", whereby a higher level of p stimulates Q_c.

By means of (3.5) we can study the increase in N_c (per unit increase in Φ^0) when Φ'' is zero, while $|\Psi''|$ is considerable. We find, *inter alia*, that the increase in N_c will exceed the increase in N_i, if $g' > (\Psi' - g')$ or, in other words, if $g'/\Psi' > 0.5$. However, in a situation where w is constant but p rises considerably "the marginal propensity to consume", g'/Ψ', may be quite low. Another observation is that, *ceteris paribus*, the higher $|G_{11}''|$ is, the less will N_i and N_c expand, as we should expect (note that G_{11}'' does not enter the second term of the denominator).

Let us now turn to a case where the *investment ceiling* is more or less *reached*, while there is still much excess capacity in the producing of C-goods —a situation which Hicks describes as "normal for a time of high activity" (See *A Contribution to the Theory of the Trade Cycle*, p. 128. Our discussion below may be compared with the discussion of Section 10 in Hicks' book.) Thus we assume that Ψ'' approaches zero, whereas the numerical value of Φ'' is considerable. In this case (3.4) reduces to:

$$(3.9) \qquad \frac{dN_i}{d\Phi^0} = \frac{\Theta\,\Phi'\,G_{11}''}{G_1'\,\Phi'' + \Theta(\Phi'')^2 G_{11}''}.$$

As both the numerator and denominator of (3.9) are negative, $dN_i/d\Phi^0$ is seen to be positive. However, (3.9) involves that, while an increase in public investment increases N_i, it has a *negative* effect on *private* investment.[2] How much (3.6) exceeds (3.9), i.e. the degree to which private investment is reduced in the case of (3.9), depends upon $|\Phi''|$. We here have the effect, suggested on p. 31 above, that, when public investment increases,

[1] But as the suggested expansion goes on, we will eventually approach the "investment ceiling" as well, whereby $|\Phi''|$ shifts strongly upwards.

[2] If private investment were to be constant, $dN_i/d\Phi^0$ would have to equal $1/\Phi'$. The right-hand side of (3.9) is obviously less than this. In (3.6), however, where we assumed that also Φ'' approaches zero, $dN_i/d\Phi^0$ approaches $1/\Phi'(N_i)$.

the I-goods-producing sector applies more of its available resources (which cannot easily be replaced) for these purposes, whereby the supply of I-goods for the private sector tends to decrease and q tends to rise.

Incidentally, Keynes' use of a *constant* relationship between private investment and the rate of interest is often criticised on the grounds that, when employment is stimulated, the consequent increase in income and in the price level will produce more optimistic price or sales expectations, so that $f(r)$ shifts *upwards*. Above we have, however, a case where employment is stimulated, but where $f(r)$ (according to our analysis) shifts *downwards*.[1]

In this case, when Ψ'', but not Φ'', approaches zero, the effect on total employment of an increase in Φ^0 is:

$$\text{(3.10)} \qquad \frac{d(N_i+N_c)}{d\Phi^0} = \frac{\Psi'}{(\Psi'-g')} \frac{\Theta\,\Phi'\,G_{11}''}{(G_1'\Phi''+\Theta(\Phi')^2 G_{11}'')}.$$

Here our "multiplier" $\Psi'/(\Psi'-g')$ is active. Comparing (3.10) with (3.7) we see, however, that the effect of an increase in public investment is now somewhat reduced. The multiplier effect is partly counterbalanced by the mentioned increase in q, which reduces the private investment to some extent.

When discussing (3.8), we suggested that, as a stability condition, the denominator has to be negative. Concerning (3.9) and (3.10), i.e. when Ψ'' approaches zero, there seems to be no problem of stability. However, we have assumed that the demand for I-goods by the I-goods producers themselves does not vary. In the case where we operate with $H_{12}'' \neq 0$, the question of stability may arise also when Ψ'' is zero.

Let us now relax the simplifying assumption that G_{12}'' (Q_c, n_c) is zero, i.e., we take into account the effects of a planned change in the labor input n_c at the point of time $t_0+\Theta$, when Q_c is delivered. Furthermore, we consider also cases where we are close to the "ceiling" in *both* sectors, so that $|\Psi''|$ and $|\Phi''|$ may both be considerable. We shall, however, still assume H_{12}'' to be zero. In this case we have to take equation (b) into account, as well as (a) and (c) on p. 32 above. Differentiating these three equations with respect to Φ^0 we get:

[1] We assume that (1.13) is written $I/p = f(r) + I_0$, so that $f(r)$ embraces private investment only.

$$(3.11) \qquad \frac{dN_i}{d\Phi^0} =$$

$$= \frac{\Theta\,\Phi'(\Psi'-g')\,[G_{11}''\,G_{22}''-(G_{12}'')^2]}{(\Psi'-g')\,\{\Theta(\Phi')^2\,[G_{11}''\,G_{22}''-(G_{12}'')^2]+G_1'\,\Phi''\,G_{22}''\}-g'\,\Psi''(re^{r\Theta}\,G_{22}''-\Phi'\,G_{12}'')}.$$

$$(3.12) \qquad \frac{dN_c}{d\Phi^0} = \frac{\Theta\,\Phi'\,g'[G_{11}''\,G_{22}''-(G_{12}'')^2]}{\text{Denominator as in (3.11)}}.$$

Both in (3.11) and (3.12) the numerator is obviously positive (cf. p. 23, where we assumed $[G_{11}''\,G_{22}''-(G_{12}'')^2]$ to be positive). As to the common denominator, the first (and rather long) term is positive. The second term may, however, be positive, which means that a negative denominator is possible. This may probably be the case when $|\Psi''|$ is relatively high, when G_{12}'' is positive (complementary factors in the production of C-goods), and at the same time $|\Phi''|$ and $[G_{11}''\,G_{22}''-(G_{12}'')^2]$ are relatively small. (These are all factors which tend to stimulate Q_c.)

It is, I think, a necessary condition for stability that the denominator is positive. If this condition is not met, a possible upward expansion will not stop before, for example, $|\Phi''|$ has shifted upwards, i.e. when the investment "ceiling" is approached (cf. our discussion concerning (3.8)).

In the case of deep depression where Ψ'' and Φ'' both approach zero, (3.11) reduces to (3.6). This means that total employment increases in accordance with the simple multiplier. The value of G_{12}'' is seen to be without any significance in this case.[1]

In the case where Ψ'', in particular, is zero, while $|\Phi''|$ is considerable, (3.11) shows that an increase in public investment actually reduces private investment, i.e. Q_c, to a certain (if small) extent. This is so because (3.11) is then smaller than $1/\Phi'$. We have the case which we discussed in connection with (3.9), where q rises while p is constant. In this case, however, the value of G_{12}'' (and the value of G_{22}'') are seen to matter (since we do not assume $G_{22}''\equiv 0$).[2]

This tendency towards a reduction in Q_c when Φ^0 increases, because q tends to rise, may be counterbalanced by an increase in p when $|\Psi''|$ is not very small. In fact, a high value of $|\Psi''|$ in relation to $|\Phi''|$ may render the effect on N_i and N_c of an increased public investment very strong, consider-

[1] This is as one should expect, because when $\Phi'' = \Psi'' \equiv 0$, p and q do not respond to an increase of public investment, whereby n_c and Q_c are constant too.

[2] Note that it does not matter whether G_{12}'' is positive or negative; only the numerical value of G_{12}'' counts.

ably stronger than the effect which the simple multiplier produces when acting by itself.

Let us also look somewhat more closely into the effect on p and q of a given rise in Φ^0. From (2.11) we find:

$$\frac{dp}{d\Phi^0}\frac{1}{p} = -\frac{\Psi''}{\Psi'}\frac{dN_c}{d\Phi^0}. \tag{3.13}$$

From (2.9) we find:

$$\frac{dq}{d\Phi^0}\frac{1}{q} = -\frac{\Phi''}{\Phi'}\frac{dN_i}{d\Phi^0}. \tag{3.14}$$

Applying (3.11) and (3.12), the relationship between the percentage increase in p and the percentage increase in q is seen to be:

$$\frac{\Psi''}{\Phi''}\frac{\Phi'}{\Psi'}\frac{g'/\Psi'}{(1-g'/\Psi')}. \tag{3.15}$$

(3.15) shows that, according to our assumptions, p and q always change in the same direction. A high level of $|\Psi''|$ in relation to $|\Phi''|$, i.e. we are closer to the "consumption ceiling" than the "investment ceiling", tends towards a stronger increase in p than in q, such as to raise p/q. But, on the other side, even if $|\Phi''|$ exceeds $|\Psi''|$ considerably, an increase in Φ^0 may raise p/q anyhow. According to (3.15), this can be the case if g'/Ψ', which we interpret as the marginal propensity to consume, is relatively high. Indeed, if g'/Ψ' is close to unity, p/q may increase even if $|\Phi''|$ is much higher than $|\Psi''|$, (i.e. Φ''/Φ' numerically much higher than Ψ''/Ψ').

Hitherto we have worked on the assumption that $H_{12}'' = 0$, which has simplified our analysis considerably. This is because when $H_{12}'' = 0$, and r is kept constant, Q_i, i.e. the *demand for I-goods by the producers of I-goods*, is invariant to changes in q and p. We shall now relax this assumption, i.e. we shall also take changes in Q_i into account.[1] To simplify we shall, however, assume that $G_{12}'' \equiv 0$.

According to (2.9), $q = w/\Phi'$. Inserting this expression into (2.6), we get:

$$H_2'(Q_i, n_i) = \Phi'(N_i).^2 \tag{3.16}$$

[1] Note that, when q changes, the producers of I-goods may according to (2.6) plan to change n_i, which affects H_1', which in turn affects Q_i.

[2] Which seems reasonable, as the producers are supposed to reckon with the same level for q and w at $t_0 + \Theta$ as at t_0.

The equations (a) and (c) on p. 32, together with (2.5) and (3.16), explain—when $G''_{12} \equiv 0$—how N_i, Q_i, N_c and n_i respond to increased public investment. Differentiating these four equations with respect to Φ^0 we get:

$$\frac{dN_i}{d\Phi^0} = \tag{3.17}$$

$$\frac{(\Psi' - g')\,\Theta\,\Phi'\,G''_{11}\{H''_{11}H''_{22} - (H''_{12})^2\}}{\{(\Psi' - g')(G'_1\Phi'' + \Theta(\Phi')^2 G''_{11}) - re^{r\Theta} g'\,\Psi''\}[(H''_{11}H''_{22} - (H''_{12})^2] + (\Psi' - g')\,\Phi'\,\Phi''\,G''_{11}H''_{12}}$$

According to our assumptions, the numerator of (3.17) is negative. We shall assume the denominator to be negative too, which is, I think, a necessary condition for stability (cf. our discussion on p. 34 and p. 37 above). It is seen that the denominator would certainly be negative in cases where $|\Psi''|$ is relatively very small and where H''_{12} is negative.

Incidentally, this suggests that we now can distinguish between *two* different possibilities of instability. First, the case which we discussed above, where $|\Psi''|$ is very high in relation to $|\Phi''|$ and $|G''_{11}|$. In this case an increase in total employment, initiated, e.g., by increased public investment, raises p, which in turn raises Q_c, whereby total employment continues to rise, which induces p to rise again, etc. So long as, *inter alia*, $|\Phi''|$ remains unchanged, this expansion will go on indefinitely.

Second, even if $|\Psi''|$ is very low, such that p is approximately constant, there is, intuitively, the possibility that an increase in Φ^0 may raise (if only slightly) q, which may induce the producers of I-goods to increase n_i and Q_i, which in turn stimulates q, whereby n_i and Q_i continue to increase, etc. One condition for such an explosion of, *inter alia*, q, must be that H''_{12} is positive, i.e. that n_i and Q_i are complementary factors of production. Note that, if Ψ'' approaches zero, the denominator of (3.17) can only be positive when H''_{12} is positive. Further, according to the denominator of (3.17) this second kind of instability[1] is, as we should expect, the more likely the lower $[H''_{11}\,H''_{22} - (H''_{12})^2]$.

Concerning the first kind of instability characterized by a relatively high value of $|\Psi''|$, we did suggest that a possible expansion may be halted because eventually $|\Phi''|$ increases.[2] Such shifts will occur when the "investment-ceiling" is approached.

As regards the above-mentioned *second kind of instability*, however, it seems more difficult to suggest an obvious reason why a possible expansion

[1] The two types of instability may, of course, both occur at the same time.

[2] As the consequent tendency for q to rise exerts a negative influence on Q_c. Cf. (2.2).

of q (and other variables) must come to a stop.[1] (Note that our assumption of a fixed r means that M is steadily increased when needed.) One may, for example, argue that when the price of I-goods rises and becomes extraordinary high during an upswing, the producers of I-goods do not expect such a favorable price situation to last. In our model, however, we work on the assumption of "*horizontal* expectations". Another possibility is that $[H_{11}'' H_{22}'' - (H_{12}'')^2]$ shifts upwards when n_i and Q_i increase.

From the equation (c) on p. 32, we get:

$$\frac{dN_c}{d\Phi^0} = \frac{g'}{(\Psi' - g')} \frac{dN_i}{d\Phi^0}. \tag{3.18}$$

Thus, if N_i increases, N_c will increase as well. As regards the total employment $(N_c + N_i)$ we consequently get that $d(N_c + N_i)/d\Phi^0$ equals $1/(1 - g'/\Psi')$ times $dN_i/d\Phi^0$.

Applying (3.13), (3.14), as also (3.18) we find the relationship between the percentage increase in p and the percentage increase in q. It is seen to be (3.15) in this case as well. Thus, the change in p/q, as a result of an increased level of public investment, is independent of the value of H_{12}''.

3 B. Shifts in the Propensity to Consume

We consider first "Mod. K.". Let us use a linear representation of the consumption function $C/p = \alpha Y + C_0$, where α and C_0 are given constants. We distinguish between two kinds of shifts in the propensity to consume. First, a shift in C_0, which, *ceteris paribus*, will change the average, but not the marginal propensity to consume. Second, a shift in α, which will, *ceteris paribus*, change both the marginal and the average propensity to consume. Actually, the effects of a change in C_0 we have already discussed (cf. the footnote on p. 30). The results are given in (3.1)–(3.3) above.

A linear consumption function together with the equations (1.10) and (1.13) give $Y = \alpha Y + C_0 + f(r)$. Assuming that r is fixed, (i.e. we operate with (1.15b)), and differentiating with respect to α, we get:

$$\frac{dY}{d\alpha} = \frac{1}{1-\alpha} Y. \tag{3.20}$$

[1] If this type of instability occurs the expansion might, according to the model, tend to go on even if we are close to, or at the point of, "full employment" globally seen. There is, however, the constraint (1.18), and, *inter alia*, the possibility that w shifts upwards. Cf. also pp. 52.

Using (1.16) and (1.17) we further get:

$$\text{(3.21)} \qquad \frac{dN}{d\alpha} = \frac{1}{1-\alpha} \frac{Y}{\Phi'}.$$

$$\text{(3.22)} \qquad \frac{dp}{d\alpha} = -\frac{1}{1-\alpha} \frac{Y\Phi''}{\Phi'} \frac{p^2}{w}.$$

(3.20) and (3.21) express a familiar Keynesian effect. The lower the marginal propensity to save, the stronger this effect is. Comparing (3.20) to (3.1) we see that $(dY/d\alpha)\,(1/Y) = dY/dC_0$. (Comparing (3.21) with (3.2) and (3.22) with (3.3), we find similar results for the variables N and p.) (3.22) shows that an increase in α will, *ceteris paribus*, raise also the price level (as Φ'' is negative). If $|\Phi''|$ is high, the price level may increase considerably even if the multiplier, $1/(1-\alpha)$, is quite low.

We turn to the extended model. We shall operate with a linear demand function for C-goods. Thus we write $g(N_i + N_c) = a(N_i + N_c) + C_0$, where a and C_0 are given constants. We shall first consider a shift in C_0, which means a change in the average, but not in the marginal, "propensity to consume" (for given total employment).

Within "Mod. K." a positive shift in the demand for C-goods, as described by a shift in the constant term C_0, and a positive shift in the level of public investment, were seen to exert the same influence on employment (and the price level). In our extended model, however, where we for one thing distinguish between the price level of C-goods and the price level of I-goods, one might—*a priori*—expect that shifts in C_0, and shifts in Φ^0, can produce different effects on total employment.[1] Intuitively, a case where p is *initially* stimulated may in its results differ from a case where q is initially stimulated.

We shall take into account that H''_{12} may be different from zero, i.e. we do not leave out the possibility that Q_i changes as a result of changes in q. On the other hand, we shall make the assumption that $G''_{12} = 0$, which simplifies our analysis considerably. Also, most of our conclusions below are probably not essentially changed in the cases where $G''_{12} \neq 0$. (Cf., e.g., the "structure" of (3.11) and (3.12) with the "structure "of (3.17) and (3.18)).

Thus, we apply the equations (3.16) and (2.5) together with (a) and (c) on p. 32, recalling that (c) is now written $\Psi(N_c) = g(N_i + N_c) = a(N_i + N_c) +$

[1] Per unit increase of C_0 or Φ^0.

$+ C_0$. These four equations form a set which explains changes in the variables N_i, N_c, Q_i and n_i. Differentiating with respect to C_0 we get:

$$\frac{dN_i}{dC_0} = \frac{\{H_{11}'' H_{22}'' - (H_{12}'')^2\} re^{r\Theta} \Psi''}{\text{Denominator as in (3.17)}}, \tag{3.23}$$

$$\frac{dN_c}{dC_0} = \frac{a}{\Psi' - a} \frac{dN_i}{dC_0} + \frac{1}{\Psi' - a}, \tag{3.24}$$

$$= \frac{(G_1' \Phi'' + \Theta(\Phi')^2 G_{11}'') \{H_{11}'' H_{22}'' - (H_{21}'')^2\} + \Phi' \Phi'' G_{11}'' H_{12}''}{\text{Denominator as in (3.17)}}.$$

We shall assume the common denominator[1] to be negative, as we did above. Our discussion (p. 39) concerning stability (of the equilibrium described by our model) concerns also (3.23) and (3.24).

The numerator of (3.23) is, according to our assumptions, negative. Thus, an increase in C_0 is seen to increase the employment of the I-goods sector. If, however, $|\Psi''|$ is very small, the increase in N_i of an increased C_0 is seen to be insignificant. Intuitively, when Ψ'' approaches zero, the increased demand for C-goods, initiated by the rise in C_0, does not raise p. Furthermore, when p remains constant, Q_c will not rise, which means that neither will q tend to rise. Thus, there is no reason to expect Q_i to rise. Consequently, in the cases where Ψ'' approaches zero, a positive shift in C_0 will *fail to affect the investment side*. However, the multiplier will naturally start its work, and continue its work, independently of whether the original increase in employment occurs in the I-goods-producing, or in the C-goods-producing sector. (Note that in the second and later "stages" of the simple multiplier process, the only expansion is in the production of *C-goods*.) In the cases where Ψ'' is zero, i.e. where the multiplier is not strengthened by any increase in p or q, we should, therefore, expect N_c to increase in accordance with the simple multiplier (independently of the values of $|\Phi''|$, $|G_{11}''|$, etc.).

As regards (3.24), the first term of the numerator is negative, while the second term may be positive in the case where H_{12}'' is positive. We shall assume that the numerical value of the first term exceeds that of the second term. Actually, this is in fact implicit in our assumption of a negative denominator.

Thus, an increase in C_0 is seen to raise N_c, as we should expect. Furthermore, we notice that the increase of N_c is now greater in relation to the increase of N_i, than in the case of (3.18). (Recall that in the case of (3.18) the

[1] By "denominator as in (3.17)" it is implied that a substitutes g'.

demand for I-goods initially increases, while by (3.24) the demand for C-goods increases initially.) Also, and in accordance with our considerations above, we see that, when Ψ'' is zero, the right-hand side of (3.24) reduces to $1/(\Psi' - a)$ or, in other words, $1/\Psi'$ times our multiplier $1/(1 - a/\Psi')$. This shows the effect on *total* employment as well, as dN_i/dC_0 is zero when Ψ'' is zero. This result is in essence the same as (3.2), which shows how total employment responds to an increase in the constant term of the consumption function within the assumptions of "Mod. K.". Thus, when we consider an increase in demand for C-goods, initiated by a positive shift in the constant term of the consumption function, the extended model gives the same simple answer as "Mod. K.", when Ψ'' (and G_{12}'') $\equiv 0$. In our discussion of the effects of a rise in the level of *public investment*, however, generally both Ψ'' and Φ'' had to approach zero for the answers of the two models to coincide.

As regards p/q, we find, applying (2.9), (2.11), (3.23) and (3.24), the relationship between the percentage increase in p and the percentage increase in q—of a rise in C_0—to be (when $\Psi'' \neq 0$):

$$(3.25) \qquad \frac{\Psi'}{\Phi''\Phi'} \frac{(G_1'\Phi'' + \Theta(\Phi')^2 G_{11}'')\{H_{11}''H_{22}'' - (H_{12}'')^2\} + \Phi'\Phi''G_{11}''H_{12}''}{re^{r\Theta}\{H_{11}''H_{22}'' - (H_{12}'')^2\}}.$$

The effect on p/q of an increase in C_0 is seen to be independent of Ψ''. Furthermore, if $|\Phi''|$ is very small, p/q will probably increase.

Will q increase more strongly in the case of an increase in Φ^0 than in the case of an increase in C_0? The input of labor needed to produce a flow of consumption goods dC_0 is dC_0/Ψ', while the input of labor needed to produce a flow of investment goods $d\Phi^0$ is $d\Phi^0/\Phi'$. Let us require that $dC_0/\Psi' = d\Phi^0/\Phi'$, i.e. that the *direct* and immediate influence on employment is the same in both cases. Applying (2.9), (3.17) and (3.23), the relationship between the increase in q as a result of a shift in Φ^0, and the increase in q as a result of a shift in C_0, is found to be ($\Phi'' \neq 0$):

$$(3.26) \qquad \frac{(\Psi' - g')\,\Theta(\Phi')^2 G_{11}''}{re^{r\Theta}\,\Psi'\Psi''}.$$

We see, for one thing, that the value of Φ'' does not affect this relationship. Furthermore, if $|\Psi''|$ is very small, q will certainly rise more in the case in which public investment increases. (In the alternative case of an increase in C_0, the rise in q approaches zero when Ψ'' approaches zero.)

Also, q tends to rise relatively strongest in the case of a rise in Φ^0, the higher $|G''_{11}|$, and the lower the rate of interest.

We discussed above the effects of a shift in C_0, the constant term of the consumption function. We shall now consider a shift in a, i.e., a shift in the marginal propensity to consume. Also in the extended model this case does not, analytically, differ much from the case of a shift in C_0. Let us, as in (3.23) above, apply equations (3.16) and (2.5), together with (a) and (c) on p. 32,[1] recalling that (c) is now written $\Psi(N_c) = a(N_i + N_c) + C_0$.

Differentiating those four equations with respect to a, we get:

$$\frac{dN_i}{da} = \frac{(N_i + N_c)\, re^{r\Theta}\,\Psi''\{H''_{11}H''_{22} - (H''_{12})^2\}}{\text{Denominator as in (3.17)}}. \tag{3.27}$$

Comparing (3.27) and (3.23), we see that dN_i/da times $1/(N_i + N_c)$ equals dN_i/dC_0. (Compare the similar results according to "Mod. K." on p. 41 above.) Further we find that $d(N_i + N_c)/da$ times $1/(N_i + N_c)$ equals $d(N_i + N_c)/dC_0$. A discussion of the effects of a shift in the marginal propensity to consume can thus be based directly upon a discussion of the effects of a shift in C_0.

3 C. An Autonomous Change in the Rate of Interest

We consider first "Mod. K.". We operate (now necessarily) with the version where r is autonomously fixed by central bank policy. The equations (1.10), (1.11) and (1.13) give $Y = F(Y) + f(r)$. Differentiating with respect to r gives:

$$\frac{dY}{dr} = \frac{f'}{1 - F'}. \tag{3.28}$$

We see that (3.28) differs from (3.1) (page 30), only by the factor f' in the numerator. Similarly, dN/dr equals (3.2) times f', and dp/dr equals (3.3) times f'.

f' is supposed to be negative. Y, N and p will consequently increase when r shifts downwards. The numerical value of f' is crucial. If f' approaches zero, a lowering of r will fail to stimulate economic activity even when the multiplier $1/(1 - F')$ is high. A low numerical value of f' may reflect a state of pessimistic expectations (during a depression). On the other hand,

[1] I.e., G''_{12} is assumed to be zero, while H''_{12} may differ from zero.

it may also, as mentioned, reflect the fact that there is no excess capacity in the I-goods-producing industry.

We turn to the extended model. We shall, as we did in Section 3 B, apply equations (3.16) and (2.5), together with equations (a) and (c) on page 32. Differentiating those four equations with respect to r, we get:

$$(3.29)\quad \frac{dN_i}{dr}=\frac{e^{r\Theta}(1+\Theta r)(\Psi'-g')\{\Phi' G''_{11}H''_{22}+\Psi'[H''_{11}H''_{22}-(H''_{12})^2]\}}{\text{Denominator as in (3.17)}}.$$

The denominator is supposed (as a stability condition) to be negative, (cf. above). The numerator is, according to our assumptions, positive. (3.29) is consequently negative, i.e., a lowering of the rate of interest will stimulate employment in the I-goods industry. This is, of course, as we should expect. Note that in (2.2) and (2.5), the only equations where r enters, a lowering of r will tend to make the left-hand side greater than the right-hand side, which, intuitively, will lead to an increase in Q_c and Q_i. A characteristic of (3.29) is that Φ'' and Ψ'' occur *in the denominator only*. We can therefore draw the conclusion that, *ceteris paribus*, the higher $|\Psi''|$ is, the stronger N_i responds to a change in r. Further, this response is the stronger, the lower, *ceteris paribus*, $|\Phi''|$ is, at least in cases where H''_{12} is negative. (All with the specification that the denominator of (3.17) is negative.) Comparing a situation of deep depression with plenty of excess capacity in both sectors (in which case, Φ'' and Ψ'' approach zero), to a situation where we are close to the consumption "ceiling" only, i.e. $|\Psi''|$ is high while $|\Phi''|$ is low, (3.29) says that the effects of a lowering of r are *comparatively low in the deep depression situation*. The same is, however, not necessarily the case if the deep depression situation is compared with a situation where the "ceiling" is approached in *both* sectors. Note that the rise in q which occurs along with a high value of $|\Phi''|$ tends to reduce Q_c. On the other hand, an increase in q may stimulate Q_i, if H''_{12} is positive. Another observation concerning (3.29) is that dN_i/dr is numerically higher, *ceteris paribus*, the higher r is.

In the case where Ψ'' and Φ'' approach zero, and where at the same time H''_{12} is zero, (3.29) reduces to:

$$(3.30)\quad \frac{dN_i}{dr}=\frac{e^{r\Theta}(1+\Theta r)(\Psi' H''_{11}+\Phi' G''_{11})}{\Theta(\Phi')^2 G''_{11}H''_{11}}.$$

It is seen that, if either $|G''_{11}|$ or $|H''_{11}|$ or both are very low, i.e. if there is only a slight tendency to decreasing returns to the single factor capital,

a lowering of the rate of interest may increase the employment of the I-goods-producing sector very strongly. (Note that the *product* of H''_{11} and G''_{11} occurs in the denominator, and that in this case q and p tend to remain constant.)

According to equation (c), p. 32, $dN_c/dr = g'/(\Psi' - g')$ times dN_i/dr. The change in N_c is thus proportional to the change in N_i. As regards the effect on total employment $(N_i + N_c)$ we thus get:

$$\frac{d(N_i + N_c)}{dr} = \frac{\Psi'}{\Psi' - g'} \frac{dN_i}{dr}. \tag{3.31}$$

This result seems quite familiar. It resembles (3.28), which says that, according to "Mod. K.", the effect on Y (and thereby on total employment) of a change in r equals the multiplier, $1/(1 - F')$, times the effect on real investment. In our extended model we interpret $\Psi'/(\Psi' - g')$, or $1/(1 - g'/\Psi')$, as expressing the multiplier, while the change in N_i expresses the change in real investment.

3 D. A Change in the Level of the Money Wage Rate

In both models the money wage w is supposed to be *exogenously* given. What are the effects of a shift in w? We consider first "Mod. K.". We operate with the alternative (1.15b), i.e., we assume that the rate of interest is (autonomously) fixed. In this case a change in w has no effects on N and Y. The fixed value of r determines I/p, which again through the multiplier determines N and Y (independently of the value of w). Therefore, by the way, w does not enter into the formulae (3.1), (3.2), (3.20), (3.21) and (3.28). The *price level*, p, is, however, dependent on the level of w. From the equation (1.17) we then see that p will change in the same direction and proportion as w.

Also our extended model possesses the specific result that a shift in w does not—when r is fixed—affect real magnitudes (as, e.g., employment), but changes the price level proportionally. That a shift in w does not change real magnitudes appears from the fact that w does not enter into the equations (a), (b) and (c) on page 32, or in (2.5) or (3.16), i.e. in the set of equations which determines N_i and N_c. It then follows from (2.9) and (2.11) that q and p must change in the same proportion as w.

Let us look at the case in which, instead of assuming that r is fixed, we operate with (1.12) and (1.15), i.e.:

$$M/p = L(Y, r), L_1' > 0, L_2' < 0,$$

where M—the nominal money supply—is autonomously given. Differentiating "Mod. K.", i.e., the equations (1.10)–(1.17), with respect to w, we get:

$$\frac{dY}{dw} = \frac{Mf'\Phi'}{p\Phi''f'M - p\Phi'w[L_1'f' + L_2'(1 - F')]}. \tag{3.32}$$

We see that in this case a shift in w *does* affect Y (and consequently N). The denominator of (3.32) is positive, according to our assumptions, while the numerator is negative. Thus, a lowering of w will increase Y. (Intuitively, p decreases, whereby M/p increases and r tends to fall, which stimulates investment.[1]) There will, however, not be any noticeable result of a lowering of w in cases where f' approaches zero, or where L_2' approaches infinity (the case of the "liquidity trap").

The expression for dp/dw has the same denominator as (3.32), the numerator being $-\Phi'p^2[L_1'f' + L_2'(1 - F')]$. Accordingly, p changes in the same direction as w, but now proportionally less.

Also in our extended model, when we apply the alternative (2.13 a) instead of (2.13 b), total employment and other real magnitudes will respond to changes in w. We shall, however, not go into this matter more closely. The formulae are, by the way, rather complex.

3 E. A Change in the Nominal Money Supply

To discuss the effects of a change in M, we cannot assume that the rate of interest is (directly) fixed. Instead we apply the liquidity function, together with the assumption that the nominal money supply is autonomously given.

Differentiating the three equations (1.10), (1.11) and (1.13) of "Mod. K." with respect to M, we get:

$$\frac{dY}{dM} = \frac{f'}{1 - F'}\frac{dr}{dM}.$$

Thus, if an increase in M lowers r appreciably, it also will raise Y appreciably, except in the case where $|f'|$ is very low.

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[1] Since this effect works through increased M/p, Keynes claims that anything which can be done by means of a possible reduction of w, he can do better by increasing M ("better" because of expectational effects).

Differentiating the equations (1.10)–(1.17) with respect to M, we get:

$$\frac{dr}{dM} = \frac{-w\Phi'(1-F')}{p\Phi''f'M - p\Phi'w[L_1'f' + L_2'(1-F')]}. \tag{3.33}$$

(3.33) is, according to our assumptions, negative. An increase in M may lower r appreciably, except when $|L_2'|$ approaches infinity, or when Φ'' approaches infinity (in which case *only* the price level will increase).

Differentiating in our extended model with respect to M yields quite complicated formulae,[1] which we shall not display here. We find, e.g., that dN_i/dM and dr/dM have opposite signs, and that assumptions which in our analysis of Section 3C make the value of dN_i/dr small, also tend to make dN_i/dM small.

3 F. Summary

Comparing the results from our two models, in regard to our questions above, we observe firstly that formally the answers of "Mod. K." are much more simple. The answers of "Mod. K." are especially simple when r is supposed to be fixed. However, the answers of the extended model are not so much more complicated, that we are unable to draw from them many general conclusions.

We find that the comparatively simple answers of "Mod. K." are in essence implicit in the more compound answers of the extended model. The former can be deduced from the latter when we make some *particular assumptions* about the *forms of the functions* in our extended model. And these particular assumptions seem in fact *realistic in a depressed economy*, where there is plenty of excess capacity in the production of both C-goods and of I-goods. Thus, we can say that the extended model justifies—under certain circumstances—the use of "Mod. K.", where, *inter alia*, the investment side of the economy is described so summarily. However, when we discuss the effects of an increase in "effective demand" in situations where we are close to the "full employment" limit, it is, for one thing, dubious to operate with an invariable form of the "marginal efficiency of capital schedule". Also, if we more particularly want to discuss the effects of an increase in "effective demand" when we are quite close to *either* the "investment-ceiling" *or* the "consumption-ceiling", "Mod. K." is inappropriate.

[1] This illustrates how Keynes by describing, *inter alia*, the investment side so summarily is able to make his point of, e.g., the "liquidity trap" by means of such a comparatively simple analysis.

4. SOME ADDITIONAL REMARKS AS TO THE ASSUMPTIONS IN OUR EXTENDED KEYNESIAN MODEL

In this last section we shall take a second look at, and discuss more thoroughly, some of the assumptions of the model of Section 2 above. We shall also, to a certain degree, attempt to extend, or refine, our model.

As pointed out on p. 27 above, the *consumption function* which we have applied, i.e. $g(N_i + N_c)$, possesses a low degree of autonomy. One reason for this is that total employment, and not total real income, enters as an argument into the function. As an improvement, we may relate the demand for C-goods to total nominal income deflated by the price of C-goods p, i.e., we may operate with real consumption as a function of $[p\Psi(N_c) + q\Phi(N_i)]/p$, instead of (2.12) above.

However, the fact that we do not take into account the *distribution* of income between wages and profits also contributes towards a low degree of autonomy for our consumption function (and towards a low degree of autonomy for Keynes' consumption function (1.11) too). (Cf. p. 27 above.) Thus, we ought to try to improve our consumption function in this respect as well.

We may divide total real demand for consumption goods into two components, viz.: the demand for C-goods by the workers, and the demand for C-goods by the producers. The former is, we assume, a function of the total wages-bill, $(N_i + N_c)w$, deflated by p. Furthermore, we may assume the aggregate real demand for C-goods by the producers to depend upon their total nominal net income, $p\Psi(N_c) + q\Phi(N_i) - (N_i + N_c)w$, deflated by the price of C-goods p. Thus, the condition which expresses that the supply of consumption goods shall equal the demand for it, now reads:

$$(4.1) \quad \Psi(N_c) = g\,\{[(N_i + N_c)w/p],\ [\Psi(N_c) + \Phi(N_i)q/p - (N_i + N_c)w/p]\}.$$

We shall assume that g_1' and g_2' are both positive and < 1, but that the marginal propensity to consume of the workers is greater than that of the producers, i.e. $g_1' > g_2'$.

Let us now resume our discussion about the effect on employment of a

shift in the level of public investment. Recalling that $p = w/\Psi'(N_c)$ and that $q = w/\Phi'(N_i)$, according to (2.11) and (2.9) respectively, (4.1) becomes:

$$(4.2)\quad \Psi(N_c) = g\,\{[(N_i + N_c)\Psi'(N_c)], [\Psi(N_c) + \\ + \Phi(N_i)\Psi'(N_c)/\Phi'(N_i) - (N_i + N_c)\Psi'(N_c)]\}.$$

We assume, in order to simplify, that H''_{12} and G''_{12} are both zero (for all actual values of Q_i, n_i, Q_c and n_c). In this case changes in N_i and N_c are explained by help of (a) on p. 32 and (4.2). Differentiating these two equations with respect to Φ° we get:

$$(4.3)\qquad \frac{dN_i}{d\Phi^0} =$$

$$\frac{\Theta(\Phi')^2 G''_{11}\{(g'_1 - 1)\Psi'\Phi' + g'_2\Phi\Psi'' + (g'_1 - g'_2)(N_i + N_c)\Phi'\Psi''\}}{\{G'_1\Phi'' + \Theta(\Phi')^2 G''_{11}\}\Phi'\{(g'_1 - 1)\Psi'\Phi' + g'_2\Phi\Psi'' + (g'_1 - g'_2)(N_i + N_c)\Phi'\Psi''\} + \\ + re^{r\Theta}\Psi'\Psi''[g'_1(\Phi')^2 - g'_2\Phi\Phi'']}.$$

$$(4.4)\qquad \frac{dN_c}{d\Phi^0} = \frac{-\Theta\Phi'\Psi' G''_{11}[g'_1(\Phi')^2 - g'_2\Phi\Phi'']}{\text{Denominator as in (4.3)}}.$$

Comparing (4.3)–(4.4) with (3.4)–(3.5) we see that our more thorough description of the consumption side has complicated our formulae considerably. Though, in the case of a deep depression, where *both* Ψ'' *and* Φ'' *approach zero*, we see that (4.3) reduces to $dN_i/d\Phi^0 = 1/\Phi'$. Thus, we get here the same conclusion as we got in the case of (3.6). The conclusion is that an expansion of public investment, $d\Phi^0$, increases N_i just as much as is needed for the said expansion, thus leaving the production of I-goods for the private sector unaffected. (Cf. p. 33 above.)

Further, when Ψ'' and Φ'' approach zero, (4.4) reduces to $dN_c/d\Phi^0 = -g'_1/(1 - g'_1)$ times $1/\Phi'$. The effect on *total* employment therefore approaches:

$$(4.5)\qquad \frac{d(N_i + N_c)}{d\Phi^0} = \frac{1}{1 - g'_1}\,\frac{1}{\Phi'}.$$

The conclusion (4.5) resembles (3.2) and (3.7). We have, in fact, again described the result of the simple multiplier process.

As our consumption function (4.1) distinguishes between the marginal propensity to consume of the workers, g'_1, and that of the producers, g'_2, one may ask why *only* g'_1 should appear in (4.5). Intuitively, when $|\Psi''|$ and $|\Phi''|$ are very small, an increase in Φ^0 induces p and q to rise but very

slightly. According to (2.11), $p\Psi'(N_c) = w$, i.e. a producer does not earn anything on his marginal worker. Now, when a (however small) rise in p occurs, $p\Psi'(N_c)$ will, *ceteris paribus*, tend to exceed w. The producers of C-goods are then supposed to increase N_c untill the equality between marginal income and outlay is restored. The increase in the net income of the producers, however, is in this case very small, while the increase in N_c and in the wages-bill may be considerable. Similarly, with a (however small) increase in q the producers of I-goods will, according to (2.9), raise N_i. They thereby in this case raise their own net income only slightly, while N_i and the wages-bill may increase considerably. Thus, this suggests that when Ψ'' and Φ'' both approach zero, an increased level of employment, caused, for example, by an increase in Φ^0, is accompanied by an increase in the wages-bill, while the net income of the producers remains almost constant. In such (extreme) situations it seems therefore to be an adequate approximation to relate changes in the demand for consumption goods to changes in total real income without explicitly going into its distribution between wages and profits. The marginal propensity to consume (out of total income) is then essentially reflecting the consumption pattern of the wage earners; (i.e. if we write the consumption function $C/p = F(Y)$, where real income Y may be defined as $(\Psi(N_c) + \Phi\ (N_i)\ q/p)$, F' is approximately equal to g_1').

It is quite often claimed that it was a serious omission of Keynes' not to take into account how the distribution of income between wages and profits affect consumption spending. However, our conclusions (4.3)–(4.5) imply, as suggested, that this omission may be *less serious* when one discusses the effects of an increase in the "effective demand" in a situation of *deep depression,* than in a situation in which we are close to the "consumption ceiling" or to the "investment ceiling" or both. (Granted that we assume, *inter alia*, a "competitive market", where (in equilibrium) marginal revenue equals marginal cost.)

The fact that (4.5) agrees with (3.7) confirms the result of Section 3 above that in the case of a depression it may be legitimate to apply a "marginal efficiency of capital schedule" which does not shift.[1] We can now add that in the same circumstances it also seems an adequate approximation to operate with Keynes' simple consumption function ignor-

[1] I.e., the substitution of (4.1) for (2.12) does not affect this conclusion.—We get, by the way, (4.5) even when H_{12}'' and $G_{12}'' \neq 0$, because when Ψ'' and Φ'' are zero the pure multiplier does its work without the interference of price changes.

ing how the income is distributed between wages and profits.

Let us now look into the case where the "investment ceiling" is quite close, while there is much excess capacity in the producing of C-goods, i.e. we assume that the numerical value of Φ'' is considerable but that Ψ'' approaches zero. In this case (4.3) approaches (3.9). Thus, we get the conclusion that an increase in public investment increases N_i, but has a negative effect on private investment. (Cf. p. 35 above.) Furthermore, in the said case, the effect on *total* employment of an increase in Φ^0 approaches:

$$(4.6) \qquad \frac{d(N_i+N_c)}{d\Phi^0} = \frac{\Theta G_{11}''[(\Phi')^2 - g_2'\Phi\Phi'']}{(1-g_1')\Phi'(G_1'\Phi'' + \Theta(\Phi')^2 G_{11}'')}.$$

The expression here is seen to be somewhat more complicated than (3.10) above. Both the numerator and the denominator of (4.6) are, according to our assumptions, negative, i.e. $(N_i + N_c)$ increases when Φ^0 rises. It is seen that the effect on $(N_i + N_c)$ is, *ceteris paribus*, the stronger the closer g_1' approaches unity. Also, the higher the producers' marginal propensity to consume, g_2', the higher is $d(N_i + N_c)/d\Phi^0$. When above we compared (3.10) with (3.7), we found that in the case of (3.10) the multiplier effect was partly offset by an increase in q. (Cf. p. 36 above.) As to (4.6), however, we cannot similarly conclude that the right-hand side of it is less than $1/(1-g_1')$ times $1/\Phi'$. Intuitively, even if a rise in q tends to reduce Q_c, and thereby to reduce the increase in N_i, we now also take into account that an increase in q raises the income of the producers of I-goods, which, *ceteris paribus*, increases their demand for C-goods. (Cf. also that (4.6) corresponds with (3.10) when $g_2' \equiv 0$.)

The use of (4.1) instead of (2.12) will also qualify several other of our conclusions in Section 3. For one thing, the introduction of a more advanced consumption function gives some new substance to our discussions regarding stability. We were (Section 3) concerned with two different causes of instability, the one being the possibility that a rise in the price level of consumption goods p, initiated by e.g. increased public investment, stimulates private investment, which in turn raises consumption spending and thereby p, etc., so that no new equilibrium is reached. Referring to our discussions on p 34, it seemed that such a tendency would prevail when

the denominator of (3.8)[1] is positive, which is likely to be the case when $|\Psi''|$ is high in relation to $|G''_{11}|$ and $|\Phi''|$. Thus, if public investment increases in a situation where there are plenty of excess capacity in the capital goods producing but not in the consumption goods producing sector (and we suppose that resources are not interchangeable), one important result seems to be that the production of capital goods "explodes" towards its ceiling. Now, when the consumption function (4.1) substitutes for (2.12), we shall similarly suspect that a *negative* (or zero valued) denominator in (4.3) implies instability. However, an instable situation seems now less probable. We cannot, as we did in Section 3, conclude that a very high value of $|\Psi''|$ in relation to $|G''_{11}|$ and $|\Phi''|$ includes instability. In fact, the denominator of (4.3) may very well be positiv on the said assumptions, and this may be the case even when Ψ'' approaches $-\infty$. (Cf. that Ψ'' enters only the second term of the denominator of (3.8), while in (4.3) it enters *both* terms of the denominator).—Intuitively, whenever the economy is stimulated in a situation where $|\Psi''|$ is very high, there is a strong tendency for the price of consumption goods p to rise, whereby the real income of the wage earners $(N_i + N_c)w/p$ may tend to decline.[2] Thus, the explosive tendency of a rise in p which we stressed above, i.e. that private investment is stimulated, which again may initiate further rise in p, etc., may more or less be *counterbalanced* by the effect that a higher p tends cet. par. to lower the real income of the workers and thereby the demand for consumption goods. For example, when Ψ'' is <0, but regardless of its numerical value, instability seems unlikely when $g'_1 \equiv 1$ and $g'_2 \equiv 0$, (cf. 4.3).

The counterbalancing effect of a rise in p may in extreme cases where Ψ'' approaches $-\infty$ be so strong that there is no increase of N_c as a result of a rise in Φ^0. But, according to (4.4), N_c will as a rule increase (even when $g'_2 \equiv 0$). Consequentely, p will also increase (cf. 2.11), and thus also Q_c and thereby private investment will tend to increase. Cf. that when $\Phi'' \equiv 0$, the right hand side of (4.3) may be written $1/\Phi'$ (which describes what is needed for the expansion of Φ°) times $a/(a–b)$, where a and b are positive constants and $a > b$ in the stable case.

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[1] By (3.8) it is assumed that H''_{12}, G''_{12} and Φ'' all approach zero. We even discussed the matter on less restrictive assumptions (on p. 37 and p. 39), these discussions modified but slightly our findings by (3.8).

[2] Cf. our assumption of a constant nominal wage rate w, and cf. (2.11) and (4.1).

The application of the simple consumption function (2.12) implied a certain relationship between possible changes in the employment of our two sectors. I.e. dN_c equaled $g'/(\Psi' - g')$ times dN_i or in other words $d(N_i + N_c)$ equaled our "multiplier" $1/(1 - g'/\Psi')$ times dN_i; (cf. that we interpreted g'/Ψ' as the marginal propensity to consume). This relationship played a rather central role in our reasoning in Section 3, for one thing we could from it and from (2.11) and (2.9) fairly easily explain how a change in a parameter would affect the relationship between q and p (cf. p. 38). The equation (4.1) does not include the same simple relationship between dN_i and dN_c, i.e., the introduction of a more advanced consumption function complicates our multiplier (i.e. the expression for $d(N_i + N_c)/dN_i$). However, in the case of deep depression, when both Ψ'' and Φ'' approach zero, the multiplier was by (4.5) above seen to be as simple as $1/(1 - g_1')$; (cf. our considerations concerning why only the marginal propensity of the workers appears in this expression). More generally, in the case of (4.3) and (4.4) the "multiplier" is seen to be: 1 + the fraction between the numerators of (4.4) and (4.3). Since both numerators are positive, the multiplier will exceed 1. But in case Ψ'' or Φ'' approaches $-\infty$, the multiplier will approach respectively 1 or ∞. Both g_1' and g_2' are seen to affect the magnitude of the multiplier except when both Ψ'' and Φ'' approach zero. In the case $g_2' \quad g_1' = 1$, the multiplier approaches infinity when Ψ'' approaches zero, infinity when Φ'' approaches $-\infty$, and zero when Ψ'' approaches $-\infty$. (Recall that the said "multiplier" describes the relationship between possible changes of $(N_c + N_i)$ and N_i, and therefore not directly and necessarily the effect of a change in e.g. public investment. Actually, in cases where a change in Φ° affects private investment, the said "multiplier" is only of limited interest.)

The reader finds perhaps that "the extended model", to a certain extent, discloses Keynes' complex investment function in that it separates the demand and supply side for I-goods. He may also consider this as a necessary step towards a more comprehensive and satisfactory treatment of the investment side. However, he may perhaps object that the extended model is primitive with respect to the treatment of price expectations and the risk factor.

Our assumption concerning price expectations was that prices are expected to remain constant (at their current levels for this "week"). Cf. p. 22 and p. 24 above. Some remarks may be offered to support this assumption.

That the single producer considers future prices to be independent of how much he plans to expand his own sales is related to our assumption about a "free competitive market" where the market shares of sellers and buyers are all relatively small.

We assumed the wage level w to be exogenously given, determined by the bargains reached between employers and employed. Further, we assumed that the producers do not expect, or take into account, future shifts in w. Our assumption of "horizontal price expectation" thus primarily implies that the present relationships between p and w, and between q and w, are expected to remain constant.

We use our extended model to discuss the effects of shifts in certain parameters. If a shift in a parameter increases, for example, the price level of C-goods p, and thereby p/w, our assumption about price expectations implies that the producers of C-goods expect this increase of p/w to be lasting, or at least they act as they do so. This assumption applies to the new equilibrium situation. We shall not go into the question whether or not it applies (without modifications) to situations outside equilibrium, as *a number* of different dynamic models may be constructed to explain the transmission from the one equilibrium situation to the other.

Keynes provides, in Chapter 12, "The State of Long-Term Expectations", in his *G.T.*, an argument in favor of "horizontal price expectations". He claims (*G.T.*, p. 148): "... the facts of the existing situation enter, in a sense disproportionately, into the formation of our long-term expectations; our usual practice being to take the existing situation and to project it into the future, modified only to the extent that we have more or less definite reasons for expecting a change". (However, Keynes investment function, when formulated in a general way as $I/p = f(r)$, does not necessarily involve the particular assumption about "horizontal" price expectation. But the quotation suggests that Keynes would probably make this assumption, if he had to specify this point of his analysis more closely.)

Applying a construction of Shackle's[1] we may, however, be able to include in our extended model parameters which roughly describe the confidence in the current price situation. Referring to the expression V in (2.1), we shall now *not* assume that the producers of C-goods calculate with a *constant* net income flow of $(pG(Q_c, n_c) - wn_c)$. They reckon, we suppose, with a

[1] See G. Shackle, "Interest-rates and the Pace of Investment", *The Economic Journal*, March 1946, p. 2 and p. 11.

net income of $(pG(Q_c, n_c) - wn_c)e^{-h_c\tau}$, where $\tau > \Theta$, and where, ordinarily, $h_c > 0$. One reason for such a more cautious calculation is, we assume, anxiety that the capital goods may become obsolete at some unpredictable date because of new inventions. Thus h_c tends to be positive. The value of h_c may further be affected by the price expectations. If, for example, we are in a boom which the producers of C-goods expect to last but a short time, h_c will tend to have a high value. If, conversely, the producers of C-goods believe an inflationary movement (in p and w) is under way, h_c will tend to have a low value, and it might even be negative. Thus, we will interpret a negative shift in h_c as expressing in a sense a better "state of confidence".

Our expression V in (2.1) now becomes:

$$V = \int_{\Theta}^{\infty} (pG(Q_c, n_c) - wn_c)e^{-(r+h_c)\tau}\,d\tau - qQ_c. \tag{4.7}$$

Instead of (2.2) we now get the following condition for a maximum of V:

$$pG_1'(Q_c, n_c) = (r + h_c)\,q\,e^{(r+h_c)\Theta}. \tag{4.8}$$

As a second condition we get, as before, (2.3).

Similarly, the expression (2.4) may be altered to:

$$V_i = \int_{\Theta}^{\infty} [qH(Q_i, n_i) - wn_i]\,e^{-(r+h_i)\tau}\,d\tau - qQ_i, \tag{4.9}$$

where h_i expresses the "confidence" that the producers of I-goods have in the current price situation. It may be important to take into account that h_i can be different from h_c. Prices of I-goods vary often more strongly over the cycle than the prices of C-goods.[1] Thus, in the time of the boom h_i may exceed h_c as the producers of I-goods may be particularly concerned that the price situation, which at the moment is very favorable to them, will probably not last. (Cf. here also our stability discussion on p. 40.) A high value of h_i may prevent the type of instability caused by the demand for I-goods by the I-goods producers, i.e., when q for some reason tends to rise, the producers of I-goods may considerably increase their own demand

[1] Cf., e.g., Hicks, *A Contribution to the Theory of the Trade Cycle*, p. 128. "We may therefore take it as normal for a time of high activity to be marked by a rise in the ratio of investment goods prices to consumption goods prices, and for a period of low activity to be marked by a fall in this ratio."

for I-goods, which causes q to continue to rise, which again stimulates the demand for I-goods by the I-goods producers, etc.

Instead of (2.5) we now get the following condition for a maximum of V_i:

$$H_1'(Q_i, n_i) = (r + h_i)\, e^{(r+h_i)\Theta}. \tag{4.10}$$

As a second condition we get, as before, (2.6).

The introduction of the parameters h_c and h_i thus alters only two equations in our model. The alteration being that $(r + h_c)$ or $(r + h_i)$ substitute for r.

Consequently, an analysis of the effects of a shift in h_c or in h_i will resemble our discussion in Section 3C of the effects of a shift in r. If we assume that $h_c = h_i = h$, a shift in h, i.e., a change in "the state of confidence", has formally the same effect as a change in r.

If we assume that $H_{12}''\equiv 0$, (4.10) implies that Q_i depends only on r and h_i. A negative shift in h_i is seen to increase Q_i. From (a), (b) and (c) on p. 32 above we then can find how an increase of Q_i will stimulate N_i and N_c (these effects will be fairly similar to those of an increase of Φ^0).

When we particularly discuss the effects of a change in h_c the analysis can be somewhat simpler. If we then assume that H_{12}'' and G_{12}'' are zero, we need only take into account the equations (a) and (c) on p. 32. (On the right-hand side of (a) $(r + h_c)$ now substitutes for r.) Differentiating with respect to h_c we get:

$$\frac{dN_i}{dh_c} = \frac{(\Psi' - g')\,\Psi'\, e^{(r+h_c)\Theta}\,[1 + (r + h_c)\,\Theta]}{(\Psi' - g')\,[\Theta\,(\Phi')^2 G_{11}'' + G_1'\Phi''] - (r + h_c)\, g'\, e^{(r+h_c)\Theta}\,\Psi''}, \tag{4.11}$$

while, according to (c), dN_c/dh_c equals dN_i/dh_c times $g'/(\Psi' - g')$.

If Ψ'' is zero, dN_i/dh_c is seen to be negative. Intuitively, a negative shift in h_c means that the producers of consumption goods become more optimistic, and therefore expand their ordering of capital goods (even if p does not rise).—If, *ceteris paribus*, $|\Psi''|$ increases, the numerical value of the denominator of (4.11) decreases, whereby the numerical value of dN_i/dh_c increases. The reason for this stronger effect of, for example, a *decrease* in h_c, is that the price of C-goods now tends to increase (and the stronger the higher the numerical value of Ψ'' is). Indeed, if $|\Psi''|$ is very high in relation to $|\Phi''|$ and $|G''|$, the situation may be unstable. (As regards this question of instability, cf. our discussion of (3.8) and (3.11) above.)

When introducing the parameters h_c and h_i we can, as suggested, describe more closely the "state of confidence". We operate, though, with h_c and h_i as *exogenously* given constants. Besides, our extended model includes a "built-in accelerator", in the sense that when a certain shift in the propensity to consume or in public investment, etc., initiates an increase in total demand, this increase may tend to raise, *inter alia*, p, which stimulates the demand for I-goods.[1] However, when we also operate with the confidence factors h_c and h_i, it is possible to study, for example, how an upswing may be initiated by an exogenous shift in h_c or in h_i.

When we speak about the "built-in *accelerator*" of our extended model we refer to a kind of "accelerator principle", where the demand for I-goods is related to *changes in prices*, i.e., we do not directly refer to the more common version of it where the demand for I-goods is related to observed or expected changes in the volume of sales.[2]

Above we assumed that the producers, when deciding how much new capital they will contract for this "week", act as if they maximize the present value of their expected income on the new capital goods. (Cf. (2.1), (2.4), (4.7) and (4.9).) Thus, when a producer reckons that a new unit of capital goods increases the present value of expected income more than he has to pay for it (or, in other words, that the new capital unit is calculated to yield an interest which exceeds r), we assume that the producer will order this unit of capital. However, one may object that this analysis is not quite general. It may in certain circumstances seem still more profitable to the producer to postpone the purchase of the unit of capital in question (for example, if he expects that the price of capital goods will soon fall considerably). Also, the calculated income of a unit of capital which the

[1] In order to take this feedback effect on investment into account in "Mod. K.", we would have to imagine upward shifts in the marginal efficiency of capital schedule. In the extended model such shifts are "built in" and explained by the model.

[2] Thus, in expositions of the accelerator principle one often imagines that the producer reacts to changes in his *sales* or sales expectations. It may be suggested, by the way, that this usual exposition of the accelerator principle does not harmonize very well with an assumption of a "competitive market", where all acting units are comparatively small. Under such circumstances it may be realistic to assume that, essentially, the existing *price*-situation or price expectations govern the investment plans of the producer, since the producer reckons that, as a rule, he can sell what he wants at the existing market price. (Cf. the notion "quantity adapter".)

producer orders today may depend upon his future investment plans. In order to take such considerations into account, we might instead of (2.1) operate with:

$$(4.12) \quad V^* = \sum_{j=1}^{n} \left\{ (pG(Q_{c1} + \ldots + Q_{cj}, n_{cj}) - wn_{cj}) \int_{\Theta+j-1}^{\infty} e^{-r\tau} d\tau - qQ_{cj} e^{-r(j-1)} \right\},$$

where Q_{cj} denotes the amount of capital the producers of C-goods plan to order in "week" j, while n_{cj} is their planned input of labor in "week" $(j + \Theta)$, i.e. when they have received the Q_{cj} units of capital. We imagine that V^* is maximized with respect to all Q_{cj} and n_{cj}, $j = 1.2 \ldots n$. Thus, when the producers decide their ordering this "week", i.e. Q_{c1}, they simultaneously plan how much they will order in the next n "weeks". (As to such a more general analysis, see, for example, G. Arvidsson, op. cit., pp. 10.

If the producers reckon with *price oscillations*, for example if they expect q to fall drastically quite soon, it may be important to explain the demand for capital goods by a more general analysis as suggested by (4.12). However, if the price expectations of the producers are rather "horizontal", it seems justified to apply a more simple set-up, for example the one we applied by (2.1) and (2.4) above.

We assumed for simplicity that the period of construction and delivery of capital goods is constant, equal to Θ "weeks". Thus, q is the price of a unit of new capital delivered in Θ "weeks". If the demand for new capital goods increases strongly in a situation where there is a pressure on the facilities for producing such goods, q may tend to increase drastically according to our assumptions. Such an increase in q will, *ceteris paribus*, tend to lower the ordering of new capital goods (at least by the producers of C-goods). It may, however, be realistic to assume that an increase in the demand for capital goods in a situation where the "investment ceiling" is reached or approached will only partly result in an increase in q, but will also tend to increase the time of delivery Θ.[1] To explain this, we would have to operate with Θ as a *variable*. We could assume that the price of capital q is, within certain limits, a decreasing function of the time of delivery,

[1] This consideration suggests that it may not be realistic to assume a high value of $|\Phi''|$ even when we are very close to the I-ceiling (since dq depends upon Φ''). On the other hand, it may be claimed that postponed deliveries and increased order stocks concerning I-goods can have similar effects as those of a rise in q, namely to reduce the demand for I-goods by the C-goods producers and to stimulate it by the very producers of I-goods.

i.e. that investors may have to pay a premium for short deliveries.[1] (In that way we may also include that the producers of capital in certain circumstances stock orders.) However, such a more general set-up, where the lag between order and delivery is not treated as a data, but explained as ultimately a result of the behavior of the market participants, would have complicated our analysis of Section 3 above considerably. Also, to operate with Θ as a variable, and also with the number of stocked orders as a variable, is generally more important in dynamic than in static studies.

[1] I have tried to develope this idea, and to go into the mentioned problem, in an article "The Market for Investment Goods. An Analysis where Time of Delivery Enters Explicitly", *The Review of Economic Studies*, Feb. 1960.

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